RATIONALIST CONTINGENCY

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Abstract

The Principle of Sufficient Reason is the principle that there is an explanation for everything. While historically significant, it has largely fallen from favor in contemporary metaphysics; in full generality, it is widely held to have untenable implications. Foremost among these is necessitarianism—the claim that everything actually true is necessarily true. This paper consists of two arguments: one negative, the other positive. The first establishes that the standard argument linking the PSR to necessitarianism fails; rationalists have independent reasons to reject the argument's background assumptions. The second provides a positive reason to hold that the PSR allows for contingency; there are distinct models of facts that are each compatible with the PSR. The PSR thus allows for it to be contingent which model obtains. I conclude that, if we are to reject the PSR, it is not because it gives rise to necessitarianism.

1 Introduction

It is not entirely clear how rationalism ought to be defined. Luminaries like Leibniz, Spinoza and Du Châtelet share important commonalities—but any similarities are punctuated by sharp disagreements. Rationalists often emphasize the role of *a priori* reflection as a basis for understanding, but diverge over the nature of substance, the existence of free will, and the relationship between mind and body. Nevertheless, if we were forced to identify rationalist metaphysics with a single, unifying doctrine, we would be hard-pressed to do any better than the Principle of Sufficient Reason (the PSR): the principle that there exists a sufficient reason for absolutely everything.

The PSR continues to play a significant—if sometimes implicit—role in metaphysical inquiry. Theories that posit brute, unexplained facts are typically treated with healthy suspicion, if not outright derision. There is something deeply unsatisfying about the claim that there is no reason why the world is structured in the way that it is—a sense of instability arising from the thought that reality is fundamentally arbitrary.

Yet many contemporary metaphysicians reject the PSR—at least in full generality. It is widely believed to have untenable implications, chief of which is necessitarianism: the claim that everything actually true is necessarily true. Necessitarianism does violence to ordinary modal judgments. It seems perfectly clear that Napoleon could have won the Battle of Waterloo (though he was actually defeated)—and that it is possible for the Apollo XIII mission to have ended in disaster (though everyone actually survived). If the PSR conflicts with these sorts of judgments, perhaps we ought to reject the PSR.

[1]My thanks to David Builes, Catherine Elgin, Peter Fritz and Alex Kocurek for helpful discussions on material within this paper.

There is an argument—hereafter referred to as 'The Standard Argument'—that links the PSR to necessitarianism.[2] While versions of this argument differ in significant respects, a typical formulation is as follows.[3] Suppose, for *reductio*, that the PSR is true and that there are contingent truths. Because there is a sufficient reason for every truth, there exists a sufficient reason R for the conjunction of all contingent truths C. In order for R to be a sufficient reason for C, it must entail C; it must be necessary that if R is true, then C is true. No necessity entails a contingency, so R must itself be contingent. But if R is contingent—and if C is the conjunction of all contingencies—then R is one of C's conjuncts. R is both a sufficient reason for, and a conjunct of, C, and so is a sufficient reason for itself. But nothing is a sufficient reason for itself. Therefore, the PSR entails necessitarianism.

This path to necessitarianism depends on more than the PSR. The Standard Argument relies upon unstated assumptions concerning the modal and logical structure of sufficient reasons. I believe that philosophers have generally—perhaps even universally—failed to appreciate their gravity. In conjunction with the PSR, these assumptions not only entail necessitarianism, but are logically inconsistent. While some might interpret this inconsistency as a decisive blow to rationalist metaphysics, my own view is that rationalists face maximal pressure to reject one of these background assumptions. My first task is thus to reconstruct the Standard Argument in a manner that makes these assumptions explicit—so that they can be examined in isolation. Ultimately, there are several that the rationalist could reasonably reject—and one that is especially suspect.

But undermining the Standard Argument will not suffice; there remains a danger that some other reasoning will establish what this argument does not. The remainder of this paper provides a positive reason to hold that the PSR allows for contingent truth. There are distinct models for the structure of facts: each of which is compatible with the PSR, but which are incompatible with one another. As such, the PSR does not determine which (if any) of these models obtains. If there is a reason to reject the Principle of Sufficient Reason, it is not because it entails necessitarianism.

2 The Standard Argument

The Standard Argument relies upon the following assumptions:

i.	*The PSR*	There is a sufficient reason for every truth.
ii.	*Factivity*	If p is a sufficient reason for a true proposition q, then p is true.
iii.	*Irreflexivity*	Nothing is a sufficient reason for itself.

[2]This argument is most often associated with van Inwagen (1983, 2015) and Bennett (1984). However, versions of it predate these—see Rowe (1975). See, also, Oppy (2009); Della Rocca (2010) and McDaniel (2019).

[3]This version of the Standard Argument most closely aligns with that presented in Levy (2016).

iv.	*Comprehension*	For every condition ϕ, if some propositions satisfy ϕ, then there exists a conjunction of all (and only) the propositions that satisfy ϕ.
v.	*Strict Implication*	If p is a sufficient reason for q, then p necessitates q—i.e., $\Box(p \rightarrow q)$.
vi.	*Distribution*	Sufficient reason distributes over conjunction. If p is a sufficient reason for a conjunction of truths $q_1 \wedge q_2 \wedge ...$, then p is also a sufficient reason for q_1 and a sufficient reason for q_2, ...

The Standard Argument can be restated so as to make these assumptions explicit.[4] As before, suppose (for *reductio*) that there exists at least one contingent truth. Given *Comprehension*, there is a conjunction of all contingent truths C. Any conjunction of contingent truths is itself a contingent truth—so C is both contingent and true. The *PSR* entails that there is a sufficient reason for C—dubbed 'R.' *Factivity* entails that R is true; *Strict Implication* that R is not necessarily true. R is thus a contingent truth, and is, by definition, a conjunct of C. By *Distribution*, R is a sufficient reason for itself—which contradicts *Irreflexivity*. Therefore, there are no contingent truths.

Given an appropriate background modal logic, this argument is provably valid. This system is a classical propositional logic with the K Axiom ($\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$) and Necessitation Rule (if $\vdash p$ then $\vdash \Box p$)—with propositional quantification and a device for rigidifying the extensions of sentential operators. (This last device allows us to refer the conjunction of all propositions that are *actual* contingent truths in every possible world—including worlds in which this conjunction is false). This system is so weak that I do not see any prospect of allowing for contingency by modifying our logic. One of these assumptions must be rejected.[5]

As noted above, these assumptions entail more than necessitarianism; they are logically inconsistent. Regardless of whether there are contingent truths, there must exist *truths*.

[4]Even after explicating these assumptions, it was not immediately obvious how this argument ought to be formalized. Successful regimentation turns on subtle considerations of modal logic—see Appendix.

[5]This logic is arguably controversial in one respect. By adopting unfree quantifiers and the Necessitation Rule, it entails *necessitism*—the claim that necessarily, everything necessarily exists (see Williamson (2013)). I myself am a necessitist, and so do not find this objectionable. This particular derivation avoids some of the most controversial implications of necessitism; by operating with propositional (rather than first- or higher-order) quantifiers, it entails that every proposition exists necessarily—not that every object does. But in practice, nearly all contingentists are higher-order contingentists (see Fritz and Goodman (2016*a*,*b*, 2018)—however, see Fairchild (2024) for an example of merely first-order contingentism). If it is contingent whether Socrates exists, perhaps it is also contingent whether the proposition *Socates is wise* exists. I doubt that appealing to contingentism is the appropriate response to this puzzle. While many contingentists operate with a free logic, they often find particular instances of Universal Instantiation unobjectionable—despite denying that it holds in full generality. I can see no reason to think that the instances of this axiom within this derivation are illicit. I suspect that even contingentists will thus feel some pressure to reject one (or more) of the principles at issue.

(After all, classical logic requires there to be at least two propositions: one true, the other false). Given *Comprehension*, there exists a conjunction of all truths T. Any conjunction of truths is itself true, so T is true. The *PSR* then entails that there is a sufficient reason for T—call it R^*. *Factivity* entails that R^* is true, and the definition of T entails that R^* is one of T's conjuncts. *Distribution* then entails that R^* is a sufficient reason for itself, which contradicts *Irreflexivity*.

This inconsistency is significant for (at least) two reasons. First, the rationalist cannot respond to this dilemma by embracing necessitarianism.[6] The claim that everything actually true is necessarily true is counterintuitive; the claim that a contradiction obtains is absurd. Every philosopher committed to consistency must reject one of the assumptions at play. Second, inconsistency helps hone our attention. While *Strict Implication* seems implicit in the Standard Argument (in particular, it served to establish that the sufficient reason for the conjunction of contingencies is itself contingent) it plays no role in the derivation of inconsistency.[7] We can thus exclude it as a candidate principle to reject. This is not to deny that *Strict Implication* is controversial—nor to claim that every rationalist ought to accept some principle along these lines.[8] But we fact a contradiction that forces the rejection of another principle *regardless* of whether *Strict Implication* is true. For our purposes, we may set it aside.[9]

The rationalist thus faces a choice: to reject *Factivity*, *Irreflexivity*, *Comprehension*, or *Distribution*. Ultimately, I think that the most promising candidate for rejection is *Distribution*—though there is some reason to resist *Irreflexivity* or *Comprehension* as well. But before evaluating each of these principles, I briefly discuss that *PSR* itself. This discussion is intended to be interpretive, rather than defensive. My aim is to investigate the relationship between the *PSR* and necessitarianism; to reject the *PSR* at this stage would be to abandon this inquiry. To that end, the following remarks are merely intended to clarify what the Principle means.

2.1 The PSR

The Principle of Sufficient Reason—as I interpret it—concerns metaphysical explanation. A sufficient reason for the fact that p is a complete explanation for why p is true. If the reason that Leo XIV is Pope is that he was elected by the College of Cardinals, then a complete explanation for why Leo XIV is Pope is that he was elected by the College of Cardinals.

[6] Della Rocca (2010) suggests embracing necessitarianism as a response to the Standard Argument.

[7] See Pruss (2006) and Amijee (2020) for two philosophers who respond to the Standard Argument by rejecting *Strict Implication*.

[8] While I myself maintain that this principle is true, Bricker (2006), Schnieder (2006), Schaffer (2010), Leuenberger (2013) and Skiles (2015) have argued against principles of this sort in the context of ground.

[9] Relatedly, the background modal logic is strictly unneeded to derive inconsistency; a classical logic with propositional quantification and a truth-predicate will suffice.

I remain neutral on the nature of metaphysical explanation. I do not presuppose an account in terms of causation, modality, or ground.[10] Nothing within my argument turns on a particular conception of explanation; what matters is that sufficient reasons (whatever those might be) logically interact with the assumptions within the Standard Argument. While I doubtlessly make numerous claims that readers could disagree with, my particular conception of explanation is not among them.

The term 'every' within 'There is a sufficient reason for every truth' is intended to express absolute generality.[11] Other versions of the *PSR* are more restrictive. Some merely hold that there are sufficient reasons for facts about existence—while others hold that there is a class of autonomous facts (which are inapt for explanation) and only require heteronomous facts to be explained.[12] A fully general interpretation of the *PSR* is more demanding than any restricted version. (After all, if absolutely every fact has an explanation, then all existence and heteronomous facts are explained). If even the most demanding interpretation allows for contingency, then less demanding interpretations presumably do as well—so it is reasonable to focus our attention on the strongest version.

I will treat *is a sufficient reason for* as a binary relation; one fact explains another. By contrast, some suggest that the relation is variably polyadic—so that any number of facts can serve as sufficient reasons.[13] For example, it may be that p and q jointly form the sufficient reason for the conjunction $p \wedge q$. There is a danger that variable polyadicity dooms the Standard Argument from the outset. If the sufficient reason for the conjunction of contingencies is a plurality of facts, then the claim that this plurality is the sufficient reason for itself is strictly ungrammatical—and so cannot possibly be derived.[14] This is not the objection I wish to address; on behalf of proponents of the Standard Argument, I will treat sufficient reason as a binary relation in what follows.

I have nothing more to say regarding the interpretation of the *PSR*. I turn my attention

[10] A causal interpretation holds that event C is a sufficient reason for event E just in case the former is a full cause of the latter—see Leibniz (1991) and Pruss (2006). A modal interpretation holds that p is a sufficient reason for q just in case it necessitates q—see van Inwagen (1983). A grounding interpretation holds that p is a sufficient reason for q just in case p grounds q, where ground is a primitive relation of metaphysical dependence—see Dasgupta (2014*b*). Each of these notions has been claimed to be unintelligible or defective in some respect. For a critique of causation, see Hume (2003). For a critique of modality, see Quine (1953). For critiques of grounding, see Wilson (2014) and Fritz (2022).

[11] For a more detailed discussion of absolute generality, see section *2.4: Comprehension* below.

[12] See Shaul (2024) for a discussion of the *PSR* restricted to facts regarding existence, and Dasgupta (2016) for a version restricted to heteronomous facts.

[13] See Schnieder and Steinberg (2016). For an argument that the problem resurfaces if we allow *is a sufficient reason for* to be fully polyadic—as suggested in Dasgupta (2014*a*)—see McDaniel (2019). For a reply, see Werner (2020).

[14] That is, if the grammatical category of 'being a sufficient reason' allows for any number of facts to serve as this relation's first input—but a single fact to serve as the second—then there is no formula within our language that expresses the claim that this plurality is a sufficient reason for itself. Proof-theories only allow for the derivation of formulae, so there is no danger of deriving the claim that a plurality is the sufficient reason for itself.

to the remaining principles: *Factivity, Irreflexivity, Comprehension* and *Distribution*.

2.2 Factivity

Factivity is the principle that falsehoods do not explain why truths obtain. If p is the sufficient reason for q, then, if q is true, p is true. If it is either sunny or rainy—and the reason that it is either sunny or rainy is that it is sunny—then the proposition *it is sunny* is true.

Factivity is deeply embedded within explanatory practice. Candidate explanations are—perhaps universally—apt for rejection if their explanans is false.[15] 'India is either in Asia or Europe because it is in Europe' ought to be rejected on the grounds that India is not in Europe—and 'Porcupines are mammals because they lay eggs' ought to be rejected on the grounds that porcupines do not lay eggs.

Factivity allows for negations to serve as sufficient reasons. For example, it may be that the reason John is a bachelor is that he is not married. But, in this case, it is the true negation (rather than the false negatum) that is explanatory. Nor does *Factivity* maintain that false propositions *cannot* explain—but rather that they *do* not. A proposition p that is actually false can serve as a sufficient reason in counterfactual situations in which it is true.

It is not unreasonable to suspect that *Factivity* is inordinately restrictive. As stated, it only governs truths; it allows for there to be sufficient reasons for falsehoods. But some might maintain that metaphysical explanation demands more—perhaps only true propositions are apt to be explained. After all, the sentence 'p because q' is (at best) misleading if p is false. We could endorse a stronger principle which holds that if p is the sufficient reason for q, then both p and q are true. But while this strengthening strikes me as extremely plausible, it is not needed for the Standard Argument to succeed; we require only a weak principle.

Resisting the Standard Argument by rejecting *Factivity* does not strike me as promising. The intuitions driving this principle are strong—and I can think of no independent reason why it should fail.

2.3 Irreflexivity

Irreflexivity is the principle that nothing is a sufficient reason for itself. As with *Factivity*, this principle is embedded within our explanatory practice; a philosopher who would explain why p is true does not merely repeat the fact that p. Suppose that A and B are discussing the sufficient reason for p, and consider the following conversation.

[15] Some philosophers (e.g., Elgin (2010, 2017) and Strevens (2020)) deny epistemic analogs of *Factivity*—holding that falsehoods can epistemically explain truths. Perhaps we further our scientific understanding by reasoning with models that, while approximately true are strictly false. However, I know of no philosophers who have denied *Factivity* for metaphysical explanation.

A: What explains the fact that *p*?

B: *p*

A: I am not interested in the question of *whether p* is true—I am willing to grant that it is. What is the *reason* that *p* is true?

B: *p*

A: Let's try another gloss. The possible worlds in which *p* is true differ systematically from those in which *p* is false. This difference, we might think, explains why *p* is true in the worlds that it is. What is this difference?

B: The *p*-worlds are *p*-worlds.

A: But that's no explanation at all! Of *course* all of the *p* worlds are *p* worlds; that's true for every proposition whatsoever—and tells us nothing about the truth of *p* in particular. *Why* is it the case that *p* is true in these worlds?

B: Because *p* is true in these worlds.

A: ????

It is easy to sympathize with *A*'s frustration. There is a nagging suspicion that *B* has provided no explanation whatsoever; merely repeating the fact that *p* does not constitute an explanation for why *p* is the case. It is remarkable how widespread the commitment to *Irreflexivity* is given that it largely rests upon intuitions like this.[16]

However, unlike *Factivity*, there is at least some independent motivation for rejecting *Irreflexivity*. A rationalist might grant that sufficient reasons are *typically* irreflexive, but hold that there are also reflexive reasons—at least in special cases.

[16]Others defend *Irreflexivity* by endorsing a layered conception of reality—see deRosset (2023). Perhaps there is a gradable notion of fundamentality—where absolutely fundamental facts serve as an explanatory basis for less fundamental facts. Since no fact is more fundamental than itself, no fact explains itself. This layered conception of reality provides a motivation for *Irreflexivity*—but not one the rationalist can endorse. Central to layered structure is that some facts hold without a sufficient reason (after all, these are to be identified with the absolutely fundamental facts). The rationalist rejects facts that hold without reason, and cannot endorse this picture.

Commitment to *Irreflexivity* is not universal. Jenkins (2011) suggests that metaphysicians err in assuming that metaphysical explanation is irreflexive; perhaps the fact that Mark is in pain is both explained by—and identical to—the fact that Mark's C-fibers are firing. Rubenstein (2024) argues (convincingly, in my view) that metaphysics requires two notions of explanation: one where more fundamental entities generate less fundamental entities, and another where the nonfundamental reduces to the fundamental. Even granting that neurology explains pain, there seems to be genuine disagreement about which sort of explanation is at issue: an irreflexive dependence associated with generation, or the reflexive identity associated with reduction. Additionally, Bliss (2013, 2018) argues against *Irreflexivity* on the grounds that it tracks an epistemic—rather than a metaphysical—conception of explanation.

Some rationalists endorse specific examples of reflexivity—and so deny that *Irreflexivity* holds with generality. Perhaps, as Spinoza (1996) claimed, the fact *God exists* explains itself. Or, to use a less theistic example, it may be that the fact *there exists at least one fact* is a sufficient reason for itself. Plausibly, existential facts are explained by their witnessing instances. Since *there exists at least one fact* witnesses itself, it may also explain itself.[17]

There are also structural reasons to reject *Irreflexivity*. Any metaphysician in the business of providing explanations faces the Agrippan Trilemma.[18] There are three candidate structures for the totality of facts:

i.	*Primitivism*	There are brute facts: those that hold without a sufficient reason, and form the ultimate sufficient reasons for everything else.
ii.	*Infinitism*	There are infinite chains of reasons, such that for every fact, there exists another which serves as the sufficient reason for it.
iii.	*Circularity*	Facts form circles of explanation; for some facts F_1, F_2, ..., F_n, F_1 is a sufficient reason for F_2, F_2 is a sufficient reason for F_3, ... , F_{n-1} is a sufficient reason for F_n, and F_n is a sufficient reason for F_1.

These options are exhaustive. The binary relation *is a sufficient reason for* is either well-founded or non-well-founded. If it is well-founded, then *Primitivism* is true; if it is non-well-founded, then either *Infinitism* or *Circularity* is true.

The *PSR* precludes *Primitivism*; it denies the existence of brute facts. The rationalist must maintain that *is a sufficient reason for* is non-well-founded, and embrace either *Infinitism* or *Circularity*.

It is hardly obvious which of these is preferable—but some might maintain that we ought to reject *Infinitism*. According to *Infinitism*, questions of the form 'Why is it the case that p?' would always have an answer—but not a particularly satisfying one. Someone who was told 'The reason for p is q' could reasonably suspect that the explanatory burden has been merely shifted from p to q. Absent an adequate explanation for q, we also lack an adequate explanation for p. An infinite series of sufficient reasons merely pushes the explanatory burden further and further: indefinitely shifting the need for explanation—without ever actually providing one. It is for this reason that, in his correspondence with Leibniz, Clarke (1716) claimed "A chain, composed of infinitely many links, can have no dependence unless it hangs upon something independent" and that Schaffer (2009)[pg. 364] claimed that, in infinite series of grounding, "[r]eality is forever deferred, never achieved." While not every rationalist rejects *Infinitism*, those that do must embrace *Circularity*.

The path from accepting *Circularity* to rejecting *Irreflexivity* is short: we need only

[17]This puzzle was first noted in Fine (2012). It is not at all clear that the appropriate response to this is to reject *Irreflexivity*. For other responses, see Correia and Schnieder (2012), Litland (2015) and Fritz (2021).

[18]The Agrippan Trilemma named by Sextus Empiricus after the skeptic Agrippa. While originally presented as an argument for skepticism, a metaphysical analog of the trilemma applies to the *PSR*.

assume that *is a sufficient reason for* is transitive. If F_1 is a sufficient reason for F_2 and F_2 is a sufficient reason for F_3, then F_1 is a sufficient reason for F_3.[19] Collectively, *Circularity* and *Transitivity* entail that some facts are sufficient reasons for themselves.

The upshot is this: as with *Factivity*, *Irreflexivity* is embedded within explanatory practice. However, there is room for the rationalist to object—by appealing either to specific examples of reflexive reasons or broader structural considerations.

2.4 Comprehension

Comprehension is the principle that, if any proposition satisfies condition ϕ, then there exists a conjunction of all propositions that satisfy condition ϕ. For example, if the proposition *Socrates is wise* involves wisdom, then there exists a conjunction of all propositions that involve wisdom.

Comprehension fails for numerous formal systems—including classical, first-order logic. Within this language, every well-formed-formula has a finite length; there are no infinitely long conjunctions. If infinitely many propositions satisfy ϕ, there does not exist a conjunction of all propositions that satisfy ϕ.

This limitation is especially significant given the role *Comprehension* plays within the Standard Argument. It serves to generate the conjunction of all contingencies (on one formulation) and the conjunction of all truths (on the other). Arguably, there are infinitely many of each. After all, if it is contingently true that there are precisely n concrete objects, it is presumably also contingently true that there are not $n + 1$ concrete objects, not $n + 2$ concrete objects, etc.[20] In order to express the relevant instances of *Comprehension*, we must expand our expressive resources.

A natural way to overcome this limitation is to introduce propositional quantifiers into our language. Rather than explicitly conjoining infinitely many ϕ-propositions, we assert that every proposition that satisfies ϕ is true:[21]

$$\forall p(\phi(p) \rightarrow p)$$

In particular, the conjunction of all contingencies is to be represented as:

$$\forall p((p \wedge \neg\Box p) \rightarrow p)$$

[19] Proof: by induction on length of circularity. There are arguments in the literature that put some pressure on transitivity. Schaffer (2012) provides counterexamples to the transitivity of ground—though it is not immediately obvious how these examples translate to the case of sufficient reason. See Litland (2013) for a reply.

[20] Trivially, if there are infinitely many contingent truths, then there are infinitely many truths. While Frege (1892) famously held that there is but a single truth (so that all true sentences have the same semantic value) the overwhelming consensus is that there are infinitely many. After all, there are infinitely many modal profiles, which correspond to different propositions (via Leibniz's Law).

[21] Alternatively, we could introduce infinitary quantifiers into our language directly; which could either be defined in terms of propositional quantifiers, or else taken to abbreviate the infinitely many conjuncts.

And—with the use of a truth predicate T—the conjunction of all truths is to be represented as:

$$\forall p(T(p) \rightarrow p)$$

Initially, *Comprehension* might seem tangential to the Standard Argument. It directly concerns neither the *PSR* nor modality. Once we adopt a language with sufficient expressive power, it would not be unreasonable to simply include *Comprehension* within our background logic—and to focus attention on other, substantive assumptions. However, perhaps the most significant challenge to the Standard Argument (in Levy (2016)) concerns this principle, so it warrants closer scrutiny.

Levy's objection relies upon the notion of *indefinite extensibility*. A property F is said to be indefinitely extensible iff, for any collection of objects that satisfy F, there exists an F which is not a member of that collection.[22] For example, the property *is an ordinal number* is indefinitely extensible, since given any collection of ordinals, a further ordinal can always be found.

In particular, Levy argues that the contingent truths are indefinitely extensible. Given any collection of contingent truths, there exists a contingent truth which is not a member of that collection.[23] If no collection of contingencies is 'complete' in this sense, it may be impossible to generate the conjunction of every contingent truth. (After all, given any conjunction of contingencies that we formed, another contingency could always be found).

Denying *Comprehension* comes with a significant cost. In classical systems with propositional quantifiers—like the simply-typed λ-Calculus—*Comprehension* is a theorem. Rejecting this principle thus comes close to rejecting classical logic itself.[24] A standard axiom of propositional identity is:

$$\forall p(\phi(p) \rightarrow p) = \forall p(\phi(p) \rightarrow p)$$

Every proposition is identical to itself, so the proposition *every ϕ-proposition is true* is identical to itself. Existential generalization then entails:

$$\exists q(q = \forall p(\phi(p) \rightarrow p))$$

There exists a proposition which holds that every ϕ-proposition is true.[25] The restriction to nonempty ϕ is strictly gratuitous; such a proposition exists for every ϕ.

[22]There are numerous precisifications of indefinite extensibility in the literature—see Uzquiano (2015). Some depend upon the interactions of extensions with modals, along the lines that, for any collection of objects that satisfy F, it is *possible* for there to exist another F—as in Builes (2022).

[23]Levy proposes—yet ultimately rejects—another objection to *Comprehension*, namely that there are 'too many' contingent truths to consider in a totality.

[24]For a discussion of the pressure indefinite extensibility places on classical logic, see Hall (2020).

[25]Here, formulae with ϕ are treated as schemata—but if we operate with the full power of higher-order logic, this need not be schematic. Rather, we could replace ϕ with a free variable ranging over sentential operators—and generalize the result of our theorem, thereby deriving $\forall X \exists q(q = \forall p(X(p) \rightarrow p))$.

Advocates of indefinite extensibility often point to inconsistencies arising from comprehension principles in other domains—like naïve set theory.[26] When confronted with the Russell Paradox, one might suspect that the reason there is no 'set of all sets' is that, for any collection of sets, it is possible to generate a further set which is not a member of that collection. But it is far from obvious how these inconsistencies impact *Comprehension*—which does not face any contradiction of this sort. As noted above, *Comprehension* is a theorem of the simply-typed λ-Calculus. The simply-typed λ-Calculus is provably sound.[27] Therefore, *Comprehension* is consistent.

Given *Comprehension*'s consistency—and the fact that it follows from eminently plausible principles of self-identity and existential generalization—the most promising path for the extensibilist is not to deny *Comprehension*, but rather to claim that it does not express what it is intended to. Along these lines, an extensibilist might claim that, while the proposition *all ϕ propositions are true* exists, the quantifier 'all' is restricted; some propositions fall outside of its domain.[28] Such an extensibilist effectively holds that we cannot make claims about absolute generality; assertions about 'all contingent truths' are such that there are contingent truths that fall outside that quantifier's scope.

Quantifier restriction undermines the Standard Argument. If the term 'every' within 'Every contingent truth is true' has a restricted scope, there may be contingent truths that the sentence does not address. At least one of these contingenties may serve as the sufficient reason for why the sentence obtains. If this is so, *Distribution* need not entail that this contingent truth is a sufficient reason for itself, and so does not violate *Irreflexivity*.

Whether this response is plausible turns on whether we can express absolute generality—an issue that remains contentious. Advocates of generality often emphasize the difficulty in denying that that we can quantify unrestrictedly in a manner that is both consistent and expresses what we want to express.[29] The sentence 'All quantifiers are restricted' includes a quantifier itself. If this quantifier is unrestricted, then the sentence is inconsistent. If it is restricted, then there are quantifiers that fall outside of its domain—quantifiers that could be unrestricted.[30] Moreover, while set-theoretic paradoxes are challenging, they

[26]See, e.g., Dummett (1978, 1981, 1991); Parsons (1974).

[27]For proof, see Barendregt (1984).

[28]While proponents of indefinite extensibility often deny that we can quantify unrestrictedly, this is not universal. See Builes (2022) for an example of a philosopher who endorses both infinite extensibility and absolute generality.

[29]For defenses of absolute generality, see Lewis (1991); van Inwagen (2002); Sider (2003) and Williamson (2003).

[30]Several philosophers have attempted to resolve the expressibility concern. See, e.g., Fine (2006) and Studd (2013). In my view, the most successful attempt occurs in Russo (Forthcoming). Russo argues that an adequate denial of absolute generality requires the ability to make non-quantificational generalizing claims. Arguably, generalized identity fits the bill: 'To be made of water is to be made of H_2O' has generalizing implications for substances made of water, and 'To be a bachelor is to be an unmarried male' has generalizing implications for bachelors. In appropriate language, we could state 'To be a quantifier is to be a restricted quantifier'—a claim which involves identification, rather than a quantification—and yet has generalizing implications for quantifiers.

can be resolved while permitting unrestricted quantification—so long as comprehension principles are formulated carefully.

In spite of this, some rationalists might prefer to resist the Standard Argument by denying absolute generality. This denial takes a stand on a contentious debate—but such stands are sometimes unavoidable. Are there any costs that the rationalist *in particular* faces—beyond the costs that denying generality typically incurs?

One significant cost is that, without the expressive resources of absolute generality, it is challenging to state the Principle of Sufficient Reason in a manner that expresses what it ought to express.[31] The *PSR* holds that there is a sufficient reason for *every* truth. If this quantifier is unrestricted, then it is possible to make claims with absolute generality—and the extensibilist's response fails. (After all, the rationalist cannot maintain that quantifiers are restricted, yet the quantifier in the statement of the Principle of Sufficient Reason is unrestricted). Alternatively, if this quantifier is restricted, then there are truths that fall outside its domain. If this is so, then the *PSR* allows there to be brute facts—those that hold without reason. So long as these facts fall outside the scope of the *PSR*'s quantifier, they may hold for no reason at all, and the sentence 'There is a sufficient reason for every fact' would remain true. But, presumably, this is *precisely* what the *PSR* was intended to deny. Rationalists who reject absolute generality thus struggle to state their view in a manner that expresses what they want to express.

The upshot is this: a rationalist could resist the Standard Argument by rejecting *Comprehension*—but they are in a somewhat uncomfortable position if they do. *Comprehension* is provably consistent, and eminently plausible assumptions entail that it is true; rejecting it outright requires rejecting classical logic. However, if it were impossible to make claims with absolute generality, *Comprehension* would involve restricted quantification. While this undermines the Standard Argument, it also threatens the *PSR* itself, as it allows for there to be facts that hold without reason—so long as these facts are not within the scope of the *PSR*'s quantifier.

2.5 Distribution

What remains is *Distribution*: the principle that, if p is a sufficient reason for a conjunction then it is a sufficient reason for each of that conjunction's conjuncts. Ought *Distribution* be accepted, or rejected? Of course, any rationalist committed to *Factivity*, *Irreflexivity* and *Comprehension* must reject *Distribution*—on pain of inconsistency. But such a rationalist

[31] I claim that it is challenging to state the *PSR*—not that it is impossible. But the rationalist would need to work hard to find a version of the *PSR* that could be expressed. Perhaps, as in Russo (Forthcoming), they could appeal to higher-order identity to generalize without the use of quantifiers; and make absolutely general claims even if quantifiers are universally restricted. One natural way to do so would be to accept 'For p to be true is for there to exist a sufficient reason that p is true.' I could not find any rationalists who endorse this precise identification—see Builes (2024) for a rationalist who considers (yet ultimately rejects) a version of the *PSR* along these lines. I suspect that any rationalist who rejects absolute generality would incur costs if their view is to be expressible—but these may be worth the price.

would rest on firmer ground with some other reason to reject it: one independent of the Standard Argument.

Explicit discussion of *Distribution* in the literature is sparse. What little there is indicates that philosophers find it to be plausible—perhaps *so* plausible as to need no defense. Some have gone so far as to claim that it is obvious to the point of rendering it unimpeachable.[32]

On the contrary. The rationalist can reasonably claim that while *Distribution* appears plausible in the abstract, close examination reveals untenable implications. There is room to resist even finite instances of *Distribution*, but the most serious problems arise for infinite instances (that is, cases where a sufficient reason distributes across infinitely many conjuncts). These problems are not limited to isolated or contrived examples; they apply across the board. Indeed, it would be more surprising to find an example of infinitary *Distribution* that succeeds, rather than one where it fails. Both the contingency and inconsistency problems for the Standard Argument rely upon infinitary *Distribution*. The rationalist can—and, in my view, should—respond by rejecting this principle.

Begin with finite *Distribution*. It is natural to hold that every fact is the sufficient reason for its self-conjunction: p is the sufficient reason for $p \wedge p$. The standard logic of ground maintains that conjunctive facts are grounded in their conjuncts, which entails that every fact grounds its self-conjunction. If the logic of ground is to serve as any guide to the *PSR*, it is reasonable to maintain that every fact is a sufficient reason for its self-conjunction. Even apart from ground-theoretic considerations, it is hard to conceive of a more satisfactory explanation for why it is the case that $p \wedge p$ than the fact that p.[33] If this is so, *Distribution*

[32]For a claim of unimpeachability, see Pruss (2006). Pruss briefly provides one argument for *Distribution*—though does not go into great depth (presumably because he believes that *Distribution* needs no serious defense). His argument is as follows:

i. If p is the sufficient reason for a conjunction, then it has sufficient information to explain all of the conjuncts.

ii. If p has sufficient information to explain q, then p is a sufficient reason for q.

iii. Therefore, if p is a sufficient reason for a conjunction, it is a sufficient reason for the conjuncts.

There is ample room for the rationalist to resist this argument. On the one hand, every fact presumably has sufficient information to explain itself (while p may not be self-explanatory, this is not because it lacks the information needed to explain p). Rationalists who accept *Irreflexivity* ought to reject premise *ii*. Additionally, metaphysical explanation is plausibly *nonmonotonic*; if p metaphysically explains q, the conjunction $p \wedge r$ need not explain q, if r is irrelevant to q. For example, it may be that the sufficient reason for the fact *water is wet* is *H_2O is wet*—while it is not the case that the sufficient reason for the fact *water is wet* is that *H_2O is wet and grass is green.* It is challenging to see how could be nonmonotonic if premise *ii* were correct. After all, if p has sufficient information to explain q, then any conjunction with p as a conjunct also has sufficient information to explain q. For a more detailed objection to Pruss, see Blado (2023).

[33]While I find the claim that p is the sufficient reason for its self-conjunction eminently plausible, I do not mean to suggest that it is uncontroversial. There is undoubtably a close connection between a fact and its self-conjunction, but it is not clear whether this relationship is an explanation, or an identification. Theories of propositional identity that license *Idempotence*—the claim that $p = p \wedge p$—ought to reject an explanatory connection between a fact and its self-conjunction. Many conceptions of propositional identity (both coarse- and fine-grained) license *Idempotence*—see, e.g., Stalnaker (1968) and Lewis (1973) for coarse-grained views and Correia and Skiles (2019) for a fine-grained view. However, there are also well-developed views that

would entail that p is a sufficient reason for itself—in violation of *Irreflexivity*. Note that this is not the claim that there are isolated, special cases where *Irreflexivity* fails. Rather, absolutely every fact would be a sufficient reason for itself. Rationalists who hold both that p is the sufficient reason for $p \wedge p$—and that there is *any fact whatsoever* that is not the sufficient reason for itself—must reject *Distribution*.

Problems compound when *Distribution* is extended to infinitary conjunction. Recall that, within this context, the conjunction of infinitely many ϕ-propositions is represented as the claim that every ϕ-proposition is true:

$$\forall p(\phi(p) \rightarrow p)$$

Applying *Distribution* to a conjunction of this form amounts to the claim that, if p is the sufficient reason that all ϕ-propositions are true, then p is the sufficient reason that each of the ϕ-propositions is true. If we allow '$<$' to denote *is a sufficient reason for*, infinitary *Distribution* can be formalized as:

$$(p < \forall q(\phi(q) \rightarrow q)) \rightarrow (\forall q(\phi(q) \rightarrow (p < q))$$

Why should we reject this principle? Because a fact can explain why a conditional obtains without explaining why the consequent is true. It may be that p explains why the conditional, 'If q, then r' holds. But if p does not also explain *why it is the case that* q, it need not also explain why it is the case that r. Suppose that p is the sufficient reason for the claim that every ϕ-proposition is true—i.e., that $\forall q(\phi(q) \rightarrow q)$. If p is not the sufficient reason for the claim that q is a ϕ-proposition (i.e., that $\phi(q)$), it need not be the sufficient reason for q—and infinite *Distribution* is therefore false.

This problem can be made concrete with the use of examples. Of course, any particular example is bound to be controversial—there are few (if any) cases of uncontroversial sufficient reasons. But there is a ready supply of cases that are somewhat plausible. For an epistemic agent s, let us consider the totality of what s knows—and introduce a sentential operator K_s for 'is known by s.' A proposition p satisfies this operator just in case s knows that p is true. With the use of this operator, we can express the conjunction of everything that s knows as the following:[34]

$$\forall p(K_s(p) \rightarrow p)$$

reject *Idempotence*—see, e.g., Dorr (2016); Fine (2017); Elgin (2020) and Bacon (2023).

[34]It is not immediately clear that epistemic agents like us have infinite knowledge—and so whether it is necessary to represent the conjunction of everything s knows with infinitary conjunction (though see Warren (2020) for an argument that we engage in infinite cognition—and note that there are versions of epistemic closure that entail that agents know infinitely many claims). Those who have this concern could modify this example—either by treating s as an idealized agent (with infinite knowledge) or by reinterpreting the operator to mean 'is known by someone-or-other'—and supposing that there are infinitely many people, each of whom knows something that no one else knows. (Perhaps each of them know a proposition that they express with 'I am here', but the indexical 'I' ensures that they refer to different propositions with that expression).

The conjunction of everything that s knows amounts to the claim that everything s knows is true. What explains why this is the case? Quite plausibly, the factivity of knowledge; the reason everything s knows is true is that knowledge is a factive mental state. If we denote the proposition *Knowledge is Factive* with '*KF*', we have:

$$KF < \forall p(K_s(p) \rightarrow p)$$

Applying *Distribution* to this results in the following:

$$\forall p(K_s(p) \rightarrow (KF < p))$$

If s knows that p, the sufficient reason for p is that knowledge is factive.

This is *wildly* implausible. s may know that the Eiffel Tower is in Paris—but it is not the case that the reason the Eiffel Tower is in Paris is that knowledge is factive. And s may know that Jannik Sinner is the first Italian to win Wimbledon—but it is not the case that the reason Jannik Sinner is the first Italian to win Wimbledon is that knowledge is factive. While the factivity of knowledge explains why everything that s knows is true, it does not explain—for a given proposition p that s knows—why p is true.

Take another example. There is some debate over the meaning of the 'actually' operator. Kripke (1980) treats this term as a rigid designator, maintaining that there is one, privileged world, and 'actually' refers to truth at this world—even when embedded within modal contexts. By contrast, Lewis (1986) treats this as an indexical operator that refers to truth at the world of utterance.[35] But on either approach, the sentence 'Actually p' has the same truth-value as 'p' when not embedded within modal contexts. Consider the conjunction of all actual truths:

$$\forall p(Actually(p) \rightarrow p)$$

What explains why this conjunction obtains? Quite plausibly, the fact that for p to be actually true is for p to be true (at least in non modal contexts). Let us dub the proposition (i.e., the proposition *in non-modal contexts, for p to be actually true is for p to be true*) 'q'. We have, then, that q is the sufficient reason that, for every proposition p, if p is actually true, then p is true. Applying *Distribution* to this results in the following:

$$\forall p(Actually(p) \rightarrow (q < p))$$

For every p, if p is actually true, then q is the sufficient reason that p is true. But this is absurd! It actually true that Vikings used antlers as combs; it is not the case that the sufficient reason for *Vikings used antlers as combs* is *in non-modal contexts, for p to be actually true is for p to be true*. It is actually true that Napoleon was once attacked by rabbits; it is not

[35]For further discussion of the meaning of 'actual,' see Stalnaker (1999); Kratzer (2002); Russell (2008) and Yablo (2008).

the case that the sufficient reason for *Napoleon was once attacked by rabbits* is *in non-modal contexts, for p to be actually true is for p to be true*.

There is thus ample reason to reject infinitary *Distribution*. And without this principle, the Standard Argument falters. As before, *Comprehension* entails that there is a conjunction of all contingent truths—which is represented as the claim that all contingent truths are true. Given the *PSR*, there is a sufficient reason for this—which may be dubbed *R*. *R* is thus the reason for the fact that, for every proposition *p*, if *p* is a contingent truth, then *p* is true. This applies to *R* itself; *R* is the reason that, *if R* is a contingent truth, *then R* is true.[36] But although *R is in fact* a contingent truth, it need not be the *sufficient reason* that *R* is a contingent truth. So, *R* need not be the reason that *R* is true.

Parallel reasoning applies to the inconsistency. Given *Comprehension*, there is a conjunction of all truths. The *PSR* then entails that there is a sufficient reason for this conjunction: a reason why, for any *p*, if *p* is true then *p*. Let us dub this reason $R^\star$. This applies to $R^\star$ itself; it is the reason that *if* $R^\star$ is true, *then* $R^\star$. But although $R^\star$ is true (given *Factivity*), $R^\star$ need not be the sufficient reason that $R^\star$ is true—and so not a sufficient reason for itself.

Or so the rationalist might argue. But proponents of the Standard Argument might push back. Perhaps the problem with the rationalist's reasoning is the identification of infinitary conjunctions with universal conditionals. After all, *Distribution* seemed initially plausible; perhaps the reason it generates such absurd results is due to a defect in how infinitate conjunctions are represented. Perhaps infinite conjunctions should be expressed differently—in a manner where *Distribution* succeeds.

But this seems a reasonable place for the rationalist to dig in. In other contexts, infinite conjunctions are represented with *precisely* this propositional quantification. It is *extremely natural* to express the conjunction of every ϕ proposition by asserting that all ϕ propositions are true. This mode of expression is not motivated by a desire to reject *Distribution*, but rather reflects a standard expansion of first-order logic. (Indeed, it would be somewhat question-begging to respond to the rationalist inclined to reject *Distribution* that their understanding of this principle must be defective—because an adequate expression is surely true). A rationalist tempted by this response maintains that infinite conjunctions *just are* propositionally quantified conditionals (or, at least, that these conditionals are as close to infinite conjunctions as anything is). *Distribution* fails for these conditionals—and so fails for infinite conjunction.

And this is where I leave the Standard Argument. It relies upon four assumptions beyond the *PSR*: *Factivity*, *Irreflexivity*, *Comprehension* and *Distribution*. I could think of no reason to reject *Factivity*, but the other three are open to doubt. Of these, *Distribution* strikes me as the most suspect. Those who endorse *Irreflexivity* and the claim that propositions are sufficient reasons for their self-conjunctions must reject this principle—and infinitary *Distribution* leads to absurd consequences. It is not entirely clear to me that the rationalist

[36] I note that even this step is not uncontroversial. *R* might be the sufficient reason that all *p*s satisfy ϕ without being the sufficient reason that a particular proposition satisfies ϕ.

has had the final word; there remains room to dispute their formulation of infinitary conjunctions. But the grounds for rejecting *Distribution* seem strong enough that it is worth examining the positive case for rationalist contingency.

3 Contingent Structure

We ought not overstate the extent of the disagreement between necessitarians and contingentarians. For any particular fact p, both might hold that p is necessarily true. They might agree that it is necessary that $2 + 2 = 4$, for example, or that Socrates is necessarily human. In principle, they could both maintain that it is necessary that the first moon landing occurred in 1969 (though it would be somewhat less natural for them to do so). The contingentarian need only establish that there is *some contingent fact or other* to prevail. Rationalism permits contingency if it allows for even a single contingent truth.

I suspect that rationalist contingency is widespread—but for the purposes of this paper I will begin with an isolated case: contingent factual structure. There are multiple ways that the facts of the world could be ordered. These orderings are all compatible with the Principle of Sufficient Reason, yet are incompatible with one another. Because the *PSR* does not determine which of these obtains, it allows for it to be contingent how the facts of the world are structured. And because the *PSR* allows for this contingency, it does not entail necessitarianism.

Recall, from the earlier discussion of *Irreflexivity*, that there are three candidate structures for facts:

i.	*Primitivism*	There are brute facts: those that hold without a sufficient reason, and form the ultimate sufficient reasons for everything else.
ii.	*Infinitism*	There are infinite chains of reasons, such that for every fact, there exists another which serves as the sufficient reason for it.
iii.	*Circularity*	Facts form circles of explanation; for some facts F_1, F_2, ..., F_n, F_1 is a sufficient reason for F_2, F_2 is a sufficient reason for F_3, ... , F_{n-1} is a sufficient reason for F_n, and F_n is a sufficient reason for F_1.

Because rationalism rules our *Primitivism*—it does not allow for brute facts—the remaining options are *Infinitism* and *Circularity*.

The prospect of contingent structure already looms large. If both *Infinitism* and *Circularity* are compatible with the *PSR*, the rationalist could hold that it is contingent whether there are infinite chains or circles of metaphysical explanation. But some might maintain that one of these alternatives is metaphysically impossible, and so is incompatible with absolutely everything (including the *PSR*). However, each structure admits of numerous permutations. These permutations all validate the *PSR*, so the *PSR* does not determine which obtains. Let us take each of these in turn.

3.1 Infinitism

Infinitism is the claim that there are infinite chains of metaphysical explanation. While there is a sufficient reason for every fact, these reasons neither terminate nor circulate. Rather, there is an unending sequence, where each element is explained by that which precedes it.

While *Infinitism* holds that there are explanatory chains of infinite length, it does not determine how many chains there are. For the sake of simplicity, suppose that there are countably infinitely many facts—and index each of these facts with the whole numbers. There is thus a fact F_0, a fact F_1, a fact F_{-1}, etc. There are different ways that these facts could explain one another. On one model, each fact is the sufficient reason for its numerical successor; fact F_{-1} is the sufficient reason for fact F_0, fact F_0 is the sufficient reason for fact F_1, etc. On a second candidate structure, the facts 'leapfrog' within the ordering. Fact F_{-2} is the sufficient reason for fact F_0, fact F_{-1} is the sufficient reason for fact F_1, etc. On this second possibility, the even-numbered facts are sufficient reasons for other even-numbered facts, and the odd-numbered facts are sufficient reason for other odd-numbered facts.

These alternatives are not merely notational variants. They correspond to distinct ways that the world could be. On the first possibility, there exists a single, infinite explanatory chain; given any pair of facts, one is the ancestral sufficient reason of the other.[37] By contrast, on the second possibility, there are two infinite chains. Given an arbitrary pair of facts, it may be that neither is the ancestral sufficient reason of the other—should they figure in different explanatory chains.

Each alternative satisfies the rationalist's demands. Regardless of whether there is one explanatory chain or two, there exists a sufficient reason for everything. Nor are these alternatives exhaustive. Most straightforwardly, for any natural number n, there could be n infinite explanatory chains. But there are other, more exotic possibilities: ones that are also compatible with the *PSR*. Perhaps each fact 'branches'—in that every fact is the sufficient reason for two distinct facts (or for three facts, or four...). Or perhaps some facts branch while others do not. The most natural way to represent these various possibilities is with directed hypergraphs. Within a hypergraph, each 'node' corresponds to a fact, and each 'arrow' corresponds to a metaphysical explanation. As it turns out there are infinitely many hypergraphs that do not terminate in either direction.[38] There possibilities all entail that every fact has an explanation—and so all are compatible with the *PSR*. Because the Principle does not entail which of these obtains, it allows for it to be contingent which obtains—and does not give rise to necessitarianism.

[37] By 'ancestral sufficient reason' I mean the transitive closure of the relation *is a sufficient reason for*. If this relation is itself transitive, then to be an ancestral sufficient reason is to be a sufficient reason—but I strictly take no stand on transitivity.

[38] There are also infinitely many when additional restrictions for irreflexivity and transitivity are introduced. In fact, not only are there infinitely many such hypergraphs, but there are *uncountably* infinitely many. These are not isomorphic to one another, and so correspond to distinct possibilities.

It might help to provide a quasi-formal explanation of this reasoning in terms of possible-worlds semantics. Let 'p' denote the claim that there is exactly one explanatory chain, and 'q' denote the claim that there are exactly two. Because p and q are both compatible with the *PSR*, there is a possible world in which the *PSR* and p are true—as well as a possible world in which the *PSR* and q are true. That is to say, $\Diamond(PSR \wedge p)$ and $\Diamond(PSR \wedge q)$. Clearly, p and q are incompatible with one another (there cannot be both exactly one and exactly two explanatory chains simultaneously). So, there is no possible world in which both p and q are true. Thus, the possible world where the *PSR* and p are true is distinct from the possible world where the *PSR* and q are true. And because there are two distinct possible worlds, there must exist propositions that are true within one world that are false within the other—i.e., contingent truths. So, even if we only countenance worlds in which the *PSR* holds, there are contingent truths.

3.2 Circularity

Circularity is the claim that there are circles of metaphysical explanation. Like *Infinitism*, *Circularity* allows for every fact to hold for a sufficient reason. Unlike *Infinitism*, it is compatible with the existence of only finitely many facts. What it requires is that there is a sequence of facts, each of which is the sufficient reason for the next; at some point within this sequence, a fact 'doubles back', and is the sufficient reason for the fact that the sequence began with. For example, it may be that F_1 is a sufficient reason for F_2—and that F_2 is a sufficient reason for F_3—while F_3 is a sufficient reason for F_1. The simplest type of *Circularity* is *Reflexivity*: a fact which is the sufficient reason for itself. (As previously noted, if the relation *being a sufficient reason for* is transitive, *Circularity* entails *Reflexivity*).

While *Circularity* holds that there are circles of metaphysical explanation, it does not determine how many circles there are. Even if we restrict our attention to reflexive sufficient reasons, there are distinct models for factual structure that validate the *PSR*. On one, there is a single violation of *Irreflexivity*. There exists but a single fact that is the sufficient reason for itself. Alternatively, there could be two violations of *Irreflexivity*; two distinct facts could be sufficient reasons for themselves. As with *Infinitism*, these are not mere notational variants. On the first possibility, all facts share an ancestral sufficient reason; the sole fact which is the sufficient reason for itself is the ancestral sufficient reason for absolutely everything. On the second possibility, there are pairs of facts whose ancestral sufficient reasons are distinct; one's ancestral reason may be the first violation of *Irreflexivity*, while the other's is the second.

Nor are these alternatives exhaustive. Most obviously, there could be three violations of *Irreflexivity*, or four, or five, etc. And, as with *Infinitism*, branching structures reveal more alternatives still. It could be that there is a single violation of *Irreflexivity*, but this fact is only an immediate sufficient reason for one other fact. Alternatively, it could branch, and be an immediate sufficient reason for two (or more) other facts. All of these infinitely many alternatives validate the *PSR*, as there is a sufficient reason for every fact. Because the *PSR*

does not determine which alternative is true, it allows for it to be contingent which obtains. And because the *PSR* allows for this contingency, it does not entail necessitarianism.[39]

3.3 Structural Facts

Several philosophers have raised the same objection in conversation—so it is worth addressing directly. On either *Infinitism* or *Circularity*, contingency arises from factual structure; for instance, it may be contingent that there is a single infinite explanatory chain—or that there is a single circular explanation. But if the *PSR* is true, these facts must themselves hold for a sufficient reason. (That is to say, there must be a sufficient reason for why there is a single explanatory chain—or reason for why there is a single circular explanation). What could explain such facts, and wouldn't any such explanation lead to necessitarianism?

In one respect, this demand is somewhat unfair to the rationalist. Rationalists do not claim to be able to *personally provide* the sufficient reason for every fact; they merely hold that such reasons exist (though they cannot always articulate them). Even if a rationalist is unsure what precisely explains factual structure, that uncertainty need not motivate rejecting the *PSR*.

But the rationalist need not respond with quietism. There are several replies compatible with the Principle of Sufficient Reason. The most natural view, I think, is that structural facts are included among all the rest.[40] An infinitist who maintains that there is a single chain holds that *all* facts fall somewhere within it—where 'all' is unrestricted. Because every fact falls within this chain—and because there exists a fact that *there is a single chain*—this structural fact is explained by other facts within the chain. Similarly, a circularist might hold that a single fact is the sufficient reason for itself, and the ancestral sufficient reason for all others. Because this fact is the ancestral sufficient reason for *every* fact, it is the ancestral sufficient reason for the fact that *a single fact is the sufficient reason for itself*.

If this is so, *Strict Implication* entails that contingency is widespread. Not only is the fact that *there is a single chain* contingent, but every ancestral sufficient reason for this fact is contingent as well (and similarly so for the fact that *a single fact is the sufficient reason for itself*). If facts necessitate that which they explain, then, if it is contingent that there is a single chain, whatever explains why there is a single chain must also be contingent. Nothing prevents the rationalist from accepting this. If there had been two explanatory chains rather than one, infinitely many other facts would have obtained, forming a contingent sequence that explains why there are two chains.

Thus, if either *Infinitism* or *Circularity* is correct, there are multiple candidate structures

[39] We could carry out analogous modal reasoning as in *Irreflexivity*—merely by allowing 'p' to denote the claim that there is one violation of *Irreflexivity*, and 'q' to denote the claim that there are two.

[40] Another alternative, which I will not explore in depth, is that there is a hierarchy of facts. There are infinitely many 'first-order' facts about the world, as well as second-order facts about the structure of first-order facts. The rationalist cannot contend that the second-order facts are brute, but may take different views on why they hold. Perhaps they hold due to the first-order facts; or perhaps they hold in virtue of one another.

for facts—each compatible with the Principle of Sufficient Reason. Because the *PSR* does not determine which structure obtains, the rationalist may hold that it is contingent which obtains. If *Strict Implication* is true, the reasons for these contingent structural facts must themselves be contingent. But the rationalist is free to endorse this, and so rationalism allows for contingency.

4 Conclusion

The Principle of Sufficient Reason is often rejected on the grounds that it entails necessitarianism. However, the path to modal collapse depends on assumptions other than the *PSR*. Several of these are open to doubt, and one—the claim that sufficient reasons distribute over conjunctions—is particularly implausible. Moreover, the rationalist has positive grounds for maintaining that the *PSR* allows for contingency. There are numerous possible structures for facts that are compatible with the Principle of Sufficient Reason. Because the Principle does not determine which structure obtains, it allows for it to be contingent which obtains. There may be a reason for absolutely everything—and yet the world could have differed from how it actually is.

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Appendix

Numerous philosophers have claimed that the Standard Argument is valid—but I could not find an adequate formalization in the literature. Successful regimentation turns on subtle considerations of modal logic—so it is worthwhile to work through some details.

Let us begin by operating with a standard propositional modal logic with the sentential operator $\Box$—such that '$\Box p$' is interpreted as 'Necessarily p'. The only assumptions needed are the K Axiom ($\Box(p \to q) \to (\Box p \to \Box q)$) and Necessitation Rule (if $\vdash p$ then $\vdash \Box p$). We also introduce a sentential operator 'C' for 'is a contingent truth,' defined in the obvious way:

$$C(p) := p \wedge \neg\Box p$$

To formalize the Standard Argument, we suppose (for *Reductio*) that there exists at least one contingent truth: $\exists x(C(x))$. We then formulate the conjunction of all contingent truths. But how is this to be done? Presumably, if there are any contingent truths, then there are infinitely many.[41] But we cannot express infinitely large conjunctions in classical logic; all well-formed-formulae have a finite length.

This expressive limitation can be overcome in (at least) two ways: by introducing either infinitary conjunction or propositional quantification into our language. I myself find propositional quantifiers to be more elegant, so opt for that approach here—but nothing significant turns on this. With this quantification, we can represent the conjunction of all contingent truths as:

$$\forall p(C(p) \to p)$$

Formalizing the Standard Argument then requires proving that this conjunction of all contingencies is itself contingent. (This is needed to establish that the sufficient reason for this conjunction does not hold necessarily, and is therefore one of this conjunction's conjuncts). But there is a problem. This conjunction is not contingent; it is necessary. Effectively, it asserts 'All contingent truths are true.' While a somewhat natural way to express the conjunction of contingencies, it holds in every possible world. After all, in an arbitrary possible world w, everything that is contingently true within w is true within w.

What we need is some way to refer *rigidly*; we require some way to conjoin the *actual* contingent truths—and express the conjunction of *those very propositions* in every possible world. If any proposition holds contingently, this conjunction will not be necessarily, and the Standard Argument can proceed. We cannot achieve this rigidity by naming each of the infinite conjuncts—and appealing to the rigidity of names. Even with the use of infinitary conjunction, we cannot manually list each of the infinitely many conjuncts.

A natural approach would be to introduce a rigidifying operator '@' for 'in actuality.' In non-modal contexts, this operator functions as a truth-predicate. p is actually true just

[41] On at least one reading, Levy (2016) argues that there are infinitely many contingent truths.

in case p is true. But when embedded within modals, this always 'goes back' to truth within the actual world. For example, if I am actually seated, then in every possible world I am actually seated. The axioms that govern this operator are the following:

$$p \leftrightarrow @p$$

$$@p \rightarrow \Box @p$$

We might use this operator to revise the definition of contingency, so that $C^*(p) := @p \wedge \neg\Box p$. As before, we might hope to then generate a conjunction of all contingent (actual) truths, and establish the contingency of this conjunction.

This strategy is ultimately unsuccessful. The problem is that introducing an actuality operator forces the restriction of the Necessitation Rule to 'pure' formulae—those that lack this operator. Without this restriction, our background logic *will itself* entail necessitarianism.[42] Applying an unrestricted Necessitation Rule to the @ axiom would result in $\Box(@p \rightarrow p)$; necessarily, if p is actually true then p is true. This amounts to the claim that necessary truth is actual truth; in every possible world, p is true just in case p is actually true.

By itself, the restriction of Necessitation is no cause for alarm. Often, inferential rules that hold with generality in less expressive languages must be restricted when further expressive resources are introduced. Take Leibniz's Law, for example. While this holds with generality in traditional first-order languages, it must be restricted when we introduce symbols for quotation. (After all, we ought not be able to infer from *Hesperus* = *Phosphorus* and *'Hesperus' begins with 'H'* that *'Phosphorus' begins with 'H'*).

The problem is that the Standard Argument crucially relies upon an *un*restricted version of the Necessitation Rule. I was unable to formalize this argument without applying the Necessitation Rule to formulae that contain the actuality operator. The fact that this system gives rise to necessitarianism is entirely unsurprising—and has no implications whatsoever for the PSR. The background logic *is itself* necessitarian, so any assumptions (including the PSR) trivially entail necessitarianism.

In order for the Standard Argument to be substantive, we require a background logic that allows for contingency; modal collapse ought to result from our substantive assumptions, not our background logic. To that end, rather than introducing rigidity at the level of propositions, I introduce it at the level of sentential operators.[43] Intuitively, we might think of these operators as picking out the set of propositions that satisfy some criterion, then referring to that very set (with that very extension) in every possible situation. (Note, however, that we do not need to reify talk of sets as first-order entities for this to succeed).

[42] See Davies and Humberstone (1980) and Williamson (2006, 2007) for discussions of this point.

[43] An alternative approach would be to introduce plural propositional quantifiers. My thanks to Peter Fritz for this suggestion.

To that end, we can express the assumptions behind the Standard Argument as follows (letting '$<$' refer to the binary sentential relation 'is a sufficient reason for,' so that '$p < q$' asserts that p is a sufficient reason for q):

i.	*The PSR*	$\forall p(p \rightarrow \exists q(q < p))$
ii.	*Factivity*	$((p < q) \wedge q) \rightarrow p$
iii.	*Irreflexivity*	$\neg\exists p(p < p)$
iv.	*Strict Implication*	$(p < q) \rightarrow \Box(p \rightarrow q)$
v.	*Comprehension*	$\exists X\forall p(Xp \leftrightarrow \phi(p))$
vi.	*Conjunction*	$\bigwedge X := \forall p(X(p) \rightarrow p)$
vi.	*Distribution*	If $p < \forall q(C(q) \rightarrow q)$ then $\forall q(C(q) \rightarrow (p < q))$

In addition to a classical logic (with classical quantifiers), this argument relies upon the following:

i.	*K*	$\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$
ii.	*Necessitation*	If $\vdash p$, then $\vdash \Box p$
iii.	*Rigidity*	$\forall p((X(p) \rightarrow \Box X(p)) \wedge (\neg X(p) \rightarrow \Box\neg X(p)))$

The Standard Argument can then be reconstructed as follows:

i.	$\exists p(p \wedge \neg\Box p)$	Supposition (for *Reductio*)
ii.	$a \wedge \neg\Box a$	Select a to witness *i*
iii.	$\exists X\forall p(X(p) \leftrightarrow (p \wedge \neg\Box p))$	*Comprehension*
iv.	$\forall p(\mathscr{C}(p) \leftrightarrow (p \wedge \neg\Box p))$	Select $\mathscr{C}$ to witness *iii*
v.	$\mathscr{C}(a)$	*ii*, *iv*, Classical Logic
vi.	$\bigwedge\mathscr{C} \leftrightarrow \forall p(\mathscr{C}(p) \rightarrow p)$	*Conjunction*
vii.	$\forall p(\mathscr{C}(p) \rightarrow p)$	Definition of $\mathscr{C}$
viii.	$\bigwedge\mathscr{C}$	*vi*, *vii*, Classical Logic
ix.	$\neg\mathscr{C}(\bigwedge\mathscr{C})$	Supposition (for *Reductio*)
x.	$\neg\bigwedge\mathscr{C} \vee \Box\bigwedge\mathscr{C}$	*vi*, Definition of $\mathscr{C}$, Classical Logic
xi.	$\Box\bigwedge\mathscr{C}$	*viii*, *x*, Classical Logic
xii.	$\Box\forall p(\mathscr{C}(p) \rightarrow p)$	*xi*, Definition of $\bigwedge$
xiii.	$\forall p(\mathscr{C}(p) \rightarrow p) \rightarrow (\mathscr{C}(a) \rightarrow a)$	Classical Logic
xiv.	$\Box\forall p(\mathscr{C}(p) \rightarrow p) \rightarrow \Box(\mathscr{C}(a) \rightarrow a)$	*xiii*, *Necessitation*, *K*, Classical Logic
xv.	$\Box(\mathscr{C}(a) \rightarrow a)$	*xii*, *xiv*, Classical Logic
xvi.	$\Box\mathscr{C}(a) \rightarrow \Box a$	*xv*, *K*, Classical Logic
xvii.	$\Box\mathscr{C}(a)$	*v*, *Rigidity*
xviii.	$\Box a$	*xvi*, *xvii*, Classical Logic
xix.	$\bot$	*ii*, *xviii*, Classical Logic

xx.	$\mathscr{C}(\bigwedge \mathscr{C})$	*ix-xix*, Classical Logic
xxi.	$\exists p(p < \bigwedge \mathscr{C})$	*v*, *PSR* Classical Logic
xxii.	$b < \bigwedge \mathscr{C}$	Select b to witness *xxi*
xxiii.	$\Box(b \rightarrow \bigwedge \mathscr{C})$	*xxiv*, *Strict Implication*
xxiv.	$\neg\Box \bigwedge \mathscr{C}$	*xx*, Definition of $\mathscr{C}$
xxv.	$\neg\Box b$	*xxiii*, *xxiv*, *K*, Classical Logic
xxvi.	b	*xxii*, *Factivity*
xxvii.	$\mathscr{C}(b)$	*xxv*, *xxvi*, Definition of $\mathscr{C}$
xxviii.	$b < b$	*xxii*, *xxvii*, *Distribution*, Definition of $\mathscr{C}$
xxix.	$\bot$	*xxviii*, *Irreflexivity*
xxx.	$\neg\exists p(p \wedge \neg\Box p)$	*i-xxix*, Classical Logic

Relatedly (and more straightforwardly), the inconsistency of the assumptions within the Standard Argument can also be formalized. The main text refers to the existence of truths, which suggests an appeal to a truth-predicate, and the truth schema $T(p) \leftrightarrow p$. With the use of this predicate, the inconsistency is derived as follows:

i.	$\forall p(T(p) \rightarrow p)$	Classical Logic, T-schema
ii.	$\exists q(q < \forall p(T(p) \rightarrow p))$	*i*, PSR
iii.	$a < \forall p(T(p) \rightarrow p)$	Select 'a' to witness *ii*
iv.	a	*i*, *iii*, Factivity
v.	$T(a)$	*iv*, T-schema
vi.	$a < a$	*i*, *v*, Distribution
vii.	$\bot$	*vi*, Irreflexivity

Given this inconsistency, there is a much more straightforward way to establish that all truths are necessary than the initial derivation (though admittedly less faithful to the reasoning implicit in the Standard Argument); we may derive the inconsistency, and simply appeal to explosion.