UNSTRUCTURED PURITY

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Forthcoming: Noûs

Abstract

Purity is the principle that fundamental facts only have fundamental constituents. In recent years, it has played a significant (if sometimes implicit) role in metaphysical theorizing. A philosopher will argue that a fact $[p]$ contains a derivative entity, and cite Purity as a reason to deny that $[p]$ is fundamental. I argue that recent developments in higher-order logic reveal a subtle ambiguity regarding the interpretation of Purity; there are stronger and weaker versions of that principle. Justifications for Purity support only the weaker interpretation, but arguments that rely upon it only succeed if the stronger interpretation holds. Consequently, nearly every metaphysician who has invoked Purity has made a mistake, in that their inferences are not justified by their arguments.

Introduction

Fundamentality holds a special place in the hearts of metaphysicians. This is not to say that metaphysics is to be defined as the study of the fundamental. The literature on derivative phenomena like causation and personal identity is expansive—and the metaphysics of race and gender has arguably received more attention in the past few decades than at any other point in history. Nevertheless, few topics have held metaphysicians' singular focus in the manner that fundamentality has. We dream of a final theory: a complete description of the ultimate foundations of the world—the basis from which all of reality arises.

Not that we appear close to realizing this dream. Fundamentality is as vexing as it is tantalizing. Some questions seem to be unanswerable by metaphysics; they fall within the purview of the empirical sciences. But even within philosophy, there are a number of intractable puzzles. What does it take for a fact to be fundamental?[2] Is there a fundamental basis at all or are there infinite chains of dependence?[3] How does the derivative depend upon the fundamental?[4]

[1]My thanks to David Builes, Verónica Gómez Sánchez, Catherine Elgin, Peter Fritz, Ezra Rubenstein and Isaac Wilhelm for their extremely enlightening discussions of material that occurs within this paper—as well as the attendees of the session on Metaphysics & Higher-Order Metaphysics at the 2024 Eastern Division of the American Philosophical Association and the 2024 Berkeley Metaphysics Workshop.

[2]There is a lively debate over the definition of fundamentality. Independence Theorists like Schaffer (2009) and Bennett (2017) hold that to be fundamental is to not depend on anything else. Minimal Foundationalists like Tahko (2018) hold that to be fundamental is to belong to the minimal basis on which everything depends. Truthmaking Foundationalists like Heil (2003) and Cameron (2008) hold that to be fundamental is to belong to the class of truthmakers for all truths. Pragmatists like Carnap (1950) and Thomasson (2015) hold that to be fundamental is to answer to certain pragmatic needs. And Primitivists like Fine (2001) and Wilson (2014, Forthcoming) hold that the notion of fundamentality is primitive and unanalyzable.

[3]For discussions about infinite chains of dependence, see Dixon (2016) and Raven (2016).

[4]There are numerous accounts of how the derivative depends upon the fundamental. Superinternalists

One principle that often governs how metaphysicians reason about fundamentality is Purity—which holds that fundamental facts only have fundamental constituents.[5] According to Purity, if the fact [Electron e is spin-up] is fundamental, then both *electron e* and the property of *being spin-up* are fundamental. Often, Purity serves as motivation for developing a positive account. A metaphysician will argue that a fact [p] (in some domain of interest) contains a derivative entity and cite Purity as a reason to deny that [p] is fundamental—before providing a theory of what [p] depends upon.

I think that arguments of this structure are flawed. Recent developments in higher-order logic reveal a subtle ambiguity regarding the interpretation of Purity; there are stronger and weaker versions of that principle. The arguments for Purity support only the weaker interpretation, yet arguments that depend upon it succeed only if the stronger interpretation is true. As a result, metaphysicians who appeal to Purity have almost universally made a mistake. The inferences that they make are not justified by the arguments that they provide. To be clear, I do not claim that the stronger interpretation *could not* be justified, but rather that it *is not*—at least at present. As much as anything, this paper is a call to action. Metaphysicians who would invoke Purity ought to provide a reason to hold that the strong interpretation is true.

A brief remark on the formalisms that follow: when I began thinking about this topic, I expected to reason within the confines of a standard simply-typed λ-calculus. However, it quickly became clear that this language lacks the expressive power that I need. I require the ability to not only quantify over terms of arbitrary type but to quantify over the types themselves. While I will begin by operating with a simply-typed language, I will shift to a version of pure-type theory—the Calculus of Constructions—when this quantification is required. I presuppose general familiarity with simply-typed systems (and so will not dedicate space to discussing how they function) but will provide an overview of pure-type theory at the appropriate time.[6] This discussion will be brief; my aim is not to study the Calculus of Constructions but to use it.

like Bennett (2011) and deRosset (2013, 2023) hold that if a fundamental fact [p_1] grounds [p_2], then [p_1] grounds the fact that it grounds [p_2]. Grounding Essentialists like Dasgupta (2014*b*) hold that, in this case, the essences of the constituents of [p_2] ground the fact that [p_1] grounds [p_2]. Bridge Principlists like Schaffer (2017) hold that there are principles akin to metaphysical laws that ground the fact that [p_1] grounds [p_2]. And Grounding Disunionists like Sider (2020) argue that grounding facts differ in how they are grounded.

[5] Arguably, the most canonical discussion of Purity occurs in Sider (2011). However, see Fine (2010), Rosen (2010), deRosset (2013), Dasgupta (2014*a*), Raven (2016) and Litland (2017) for other defenses of Purity. For arguments against Purity, see Merricks (2013) and Barker (2023).

[6] I direct those seeking an introduction to simply-typed languages to texts dedicated to that purpose—e.g., Dorr, Hawthorne and Yli-Vakkuri (2021), Bacon (2023*a*), Bacon and Dorr (2024), Dorr (Forthcoming) and Goodman (2024). Suffice it to say that I adopt a functional, higher-order language with two basic types: a type e for entities and a type t for sentences. For any types τ_1 and τ_2, there is a type $(\tau_1 \to \tau_2)$—which is to be interpreted as a function from terms of type τ_1 to terms of type τ_2. There are infinitely many constants and variables of every type, as well as λ terms that serve to bind these variables. The logical operators are identified with constants in the standard way. I omit types when they are contextually evident or when formulae are to be interpreted as schemata with applications in every type.

Higher-Order Structure

It is not unreasonable to identify the beginning of the analytic tradition with the development of type-theory. The insights of Frege (1884), Russell (1903, 1908) and Church (1940) not only served to precisify philosophical argumentation but did so within the framework of the rigid, hierarchical structure of the theory of types. But attention to these systems waned by the second half of the 20th century. Quine (1970)'s insistence on the primacy of first-order logic—and denigration of type-theory as "set theory in sheep's clothing" (p. 66) left little use for higher-order reasoning. Fortunately, matters have changed in recent years; type-theory has become nearly compulsory in much of contemporary metaphysics.[7]

One of the most significant programs within higher-order metaphysics has been a sustained assault on accounts of structured propositions.[8] One of the central commitments of structuralism is that propositions and properties are 'built' from worldly material—in much the way that sentences and clauses are built from words.[9] The proposition *Socrates is wise* is composed of Socrates and the property of *being wise* and the proposition *Napoleon is short* is composed of Napoleon and the property of *being short*. Propositions built from different material are distinct due to their differing compositions. Correspondingly, identical propositions are all built from the same material; they have the same properties and objects contained within them. Structuralists thus endorse the Principle of Singular Extraction (the PSE) according to which if $Fa = Gb$, then $F = G$ and $a = b$.

The PSE radically conflicts with an orthodox principle of higher-order logic: β-equivalent terms co-refer.[10] For example, β-identification entails that $Fa = \lambda x.Fx(a)$. Jointly, the PSE and β-identification entail *higher-order monism*. For each type τ, all constants of type τ co-refer. This can be established as follows:

i.	$\lambda x(x = x)(a) = \lambda x(x = a)(a)$	β-identification
ii.	$\lambda x(x = x) = \lambda x(x = a)$	*i*, PSE
iii.	$\forall x(x = x)$	Classical Logic
iv.	$\forall x(x = a)$	*ii*, *iii*, Leibniz's Law
v.	$\exists y \forall x(x = y)$	*iv*, Classical Logic

This derivation is to be interpreted as a schema with applications in every type. It not only follows that there is only a single object but also that there is only a single property, only a single relation, only a single sentential operator, and, most worryingly,

[7] For significant works that employ higher-order inferences, see, e.g., Williamson (2003), Dorr (2016), Bacon and Russell (2019), Caie, Goodman and Lederman (2020), Dorr, Hawthorne and Yli-Vakkuri (2021) and Fritz and Jones (2024).

[8] See, e.g., Dorr (2016), Goodman (2017), Fritz (2022) and Elgin (2024*b*).

[9] An alternate conception of structured propositions is advanced by Bacon (2023*b*), building on Dixon (2018), according to which propositions have pictorial, rather than syntactic structure.

[10] This inconsistency was first noted in Dorr (2016)—though this particular proof is from the simplified derivation in Elgin (2024*b*). As noted below, there is more controversy over whether vacuously β-equivalent terms co-refer. At present, I am concerned with nonvacuous conversion.

only a single proposition. Because all propositions are identical (in general), $p = \neg p$ (in particular). Because p is identical to its negation, the two have the same truth-value. Higher-order monism is not only unintuitive, but inconsistent.

I myself interpret this as a reason to reject the PSE—and, by extension, structured theories of propositions. But this inconsistency could be resolved by rejecting β-identification instead.[11] This derivation establishes that the PSE and β-identification are mutually incompatible; it does not determine which principle we ought to reject.

But there is another (arguably even more serious) problem for structured propositions: one that does not rely upon β-identification.[12] The problem concerns how many propositions there are: the cardinality of the set of propositions.[13] Structuralists maintain that syntactically distinct sentences express different propositions; because 'Grass is green' differs syntactically from 'Grass is not not green,' the sentences differ in their semantic values. The problem is that, for every collection of propositions, it is possible to construct a sentence asserting that their conjunction is true. These conjunctions all differ syntactically from one another (after all, each is a conjunction comprised of different conjuncts). Due to their syntactic differences, structuralists maintain that they all express different propositions. For this reason, there is an injection from the powerset of propositions to the set of propositions; each element of the powerset is first mapped to a sentence conjoining those propositions, and this sentence is then mapped to a unique proposition. But Cantor's Theorem entails that there is no such mapping. There cannot exist an injection from the powerset of a set S to set S. So, some syntactically distinct sentences express the same proposition.

These arguments differ in their details, but their implications are the same. A proposition can be built in numerous ways; given a proposition p, we cannot determine the unique components from which p was constructed. For example, the proposition Raa could be constructed from $\lambda x.Rxx$, $\lambda x.Rxa$ or $\lambda x.Rax$. All of these could be seen as figuring within Raa; there is no fact of the matter as to which is 'the' property contained therein. Accounts that depend upon singular construction—like the structured view—are false. While much (though not all) of this discussion has centered on propositions, analogous arguments apply to facts.[14] Just as the same proposition could be constructed from different material, the same fact could be constructed from different material; when theorizing about facts, we cannot presuppose a unique method of construction.

[11] Two philosophers who have advocated rejecting β-identification are Rosen (2010) and Fine (2012), who argue that terms are grounded in, rather than identical to, their β-conversions. For a defense of β-identification, see Dorr (2016).

[12] This problem was first discovered by Russell (1903) and noted (apparently independently) by Myhill (1958)—and, for this reason, goes by the name 'The Russell-Myhill.'

[13] My reference to set theory here is purely expository—it is possible to generate this problem without reference to sets.

[14] An application of this problem to facts occurs in Fritz (2022).

Ambiguous Purity

Given that facts admit of multiple methods of construction, Purity—the principle that fundamental facts only have fundamental constituents—can be interpreted in (at least) two ways. In natural language, we might express the stronger interpretation as the claim that, if a fact is fundamental, then *every* way to construct it relies upon purely fundamental constituents—and the weaker interpretation as the claim that, if a fact is fundamental, then *at least one* way to construct it relies upon purely fundamental constituents.

An example highlights the distinction between the strong and weak interpretations. Suppose that $[Fa] = [Gb]$, that F, a and $[Fa]$ are all fundamental, but that G and b are derivative. This case (should it exist) falsifies the strong interpretation of Purity, but partially verifies the weak interpretation.[15] There is one way of constructing the fundamental fact $[Fa]$ from the fundamental—by predicating F of a. However, there is another way of constructing the same fact from the derivative—by predicating G of b.[16]

It is valuable to formalize this distinction. Those who would appeal to Purity in formal argumentation have a need to state which principle they appeal to. Moreover, while natural language uncovers a distinction between two interpretations of Purity, formal languages reveal far more. For every type τ, let us introduce a predicate *Fundamental* of type $(\tau \to t)$, with the intended interpretation that '$Fundamental(A)$' asserts that A is fundamental.[17] At a first pass, we might suggest the following:

> **Strong Purity$_1$:**
> $\forall p^t \forall X^{e \to t} \forall x^e ((Fundamental(p) \wedge Xx = p) \to (Fundamental(X) \wedge Fundamental(x))$
>
> **Weak Purity$_1$**:
> $\forall p^t (Fundamental(p) \to \exists X^{e \to t} \exists x^e (Xx = p \wedge Fundamental(X) \wedge Fundamental(x))$

According to Strong Purity$_1$, if a fact is fundamental, then all monadic, first-order properties and objects that construct that fact are fundamental. According to Weak Purity$_1$, if a fact is fundamental, then there is at least one way to construct that fact from a fundamental monadic, first-order property and fundamental object.

Both principles only apply to a single sort of predication—and lack their intended scope. For example, Strong Purity$_1$ allows for $[\forall x Fx]$ to be fundamental even if $\lambda x.Fx$ is derivative. While there is a way of constructing $[\forall x Fx]$ from derivative terms (namely, by predicating $\forall$ of $\lambda x.Fx$), this construction does not involve first-order predication and so

[15] The claim that this merely partially verifies Weak Purity is due to the fact that this principle requires that all fundamental facts—not merely $[Fa]$—be constructible from fundamentalia.

[16] Note that the weak interpretation is not the claim that at least one constituent of $[Fa]$ is fundamental but rather that there is at least one method of construction where all of the components are fundamental.

[17] Note that introducing these predicates does not itself guarantee that, for every type τ, there is a fundamental term of that type; some (and, in principle, all) of these predicates could have empty extensions. These predicates merely allow us to grammatically make claims about fundamentality.

falls outside Strong Purity$_1$'s scope. Correspondingly, Weak Purity$_1$ is too restrictive. This principle ought to state that there is *some method or other* of constructing each fundamental fact from fundamentalia—not that this construction need involve first-order predication.

We can generalize Strong Purity$_1$ into a schema with applications in every type:

Strong Purity$_2$:
$\forall p^t \forall X^{\tau \to t} \forall x^\tau ((Fundamental(p) \wedge Xx = p) \to (Fundamental(X) \wedge Fundamental(x)))$

Strong Purity$_2$ effectively generalizes over the types; for every type τ, a principle of this form holds. This quantification can only be made explicit in the meta-language. It is impossible, within the simply-typed λ-calculus, to quantify over terms of all types whatsoever. Quantifiers are constants that occur within the hierarchy of types: ones that quantify only over terms lower on that hierarchy than themselves. Any quantifier stated within the object-language would omit instances of Strong Purity that fall higher on the hierarchy than itself.

How are we to express Weak Purity? As with the generalization of Strong Purity$_1$, we might be tempted by a schematic principle:

Weak Purity$_2$:
$\forall p^t (Fundamental(p) \to \exists X^{\tau \to t} \exists x^\tau (Xx = p \wedge Fundamental(X) \wedge Fundamental(x))$

Weak Purity$_2$ is far too demanding for our purposes (indeed, even more demanding than Weak Purity$_1$). It does not state that there is some-way-or-other to construct each fundamental fact but rather that there is a method of fundamental construction *for every type*. It holds that each fundamental fact can be constructed from a fundamental first-order property, from a fundamental second-order property, from a fundamental sentential operator, and, indeed from a fundamental term of each of the infinitely many types that there are. The problem is that the schematic approach effectively *universally* quantifies over the types—but we require existential quantification.

We are running head-first into the expressive limitations of simply-typed languages.[18] In disambiguating Purity, we attempt to quantify over the types themselves: to distinguish the claim that terms of *every* type that construct fundamental facts are themselves fundamental—from the claim that terms of *some* type that construct fundamental facts are themselves fundamental. These quantifiers cannot be stated in simply-typed languages. The right thing to do, when confronting this limitation, is to shift to a language where they can be.

The Calculus of Constructions

One language that allows for type-quantification is the Calculus of Constructions—a version of pure-type theory. Constructing formulae is somewhat more cumbersome in

[18]For a more detailed discussion of limitations of this sort, see Wilhelm (Forthcoming*b*,F).

this language than in simply-typed systems, so it is worthwhile to briefly characterize how it operates.

In simply-typed languages, types serve to mark the syntactic categories of various terms. While they occur within formulae, there is a sense in which they are semantically idle; claims are not made about the types themselves. By contrast, within the Calculus of Constructions, claims are made explicitly about types. We can assert that e (the type of entities) is a type, that it is self-identical, and that it is the grammatical category of the singular term 'a.' In order to express these claims, we require additional grammatical categories. Two categories (called 'sorts') are especially significant: $\star$, which is the grammatical category of types, and $\square$, which is the grammatical category of (some) grammatical categories.

As in simply-typed languages, the Calculus of Constructions has infinitely many variables and constants of type e and t—and of the derivative types constructed from them. It also has infinitely many variables for the types themselves. (Notably, however, there are no variables that range over both types and ordinary terms). We thus require a device for variable-binding. This is accomplished with the operator Π—often in strings of the following form:

$$\Pi x : A$$

The subexpression '$x : A$' asserts that the term x has grammatical category A. For example, '$a : e$' asserts that a has the grammatical category e (i.e., entities)—and '$e : \star$' asserts that e has the grammatical category of $\star$ (i.e., types).[19] Note, from these examples, that types can occur on either side of ':'. When they occur on the right, they perform a similar function as they do within λ—indicating the grammatical category of the term on the left. However, when they occur on the left, they figure within the 'object language'; claims are made about the types themselves.

The Π operator is used to construct derivative grammatical categories in the following way.[20] For (almost all) grammatical categories A and B, there is a category:

$$\Pi x : A.B$$

[19] In what follows, I typically refer to variables ranging over terms with English letters beginning with 'x'—and variables ranging over types with Greek letters.

[20] More precisely, the language of the Calculus of Constructions is given by the following:

1. $\star$ and $\square$ are constants where $\star$ has grammatical category $\square$.
2. e and t are constants that both have grammatical category $\star$.
3. For any string of symbols A and B such that A is of category s and B is of category s', for every variable x of category A, the string '$\Pi x : A.B$' is of category s'.
4. For any string of symbols A of grammatical category s for some sort s, there are infinitely many constants and variables of category A.

A quick point about syntax. In line 4, note the restriction to an s that is a *sort*. This prevents terms like 'p' from functioning as grammatical categories themselves, as the string 'p' is of grammatical category t, which is a type (and not a sort).

This is the grammatical category of terms that combine with expressions of category A in order to generate expressions of category B. For example, the category of monadic, first-order predicates is given by: $\Pi x : e.t$—rather than $e \rightarrow t$ (as in λ). As it turns out, the Calculus of Constructions encodes all of the grammatical categories and terms of simply-typed languages; there is an isomorphic replica of the simply-typed λ-calculus within the Calculus of Constructions.[21] The categories of some of the most logically significant terms are the following:

1. $\neg$ is of category $(\Pi x : t.t)$
2. $\wedge, \vee, \rightarrow$, and $\leftrightarrow$ are of category $(\Pi x : t.(\Pi y : t.t))$
3. $=$ is of category $(\Pi \alpha : \star.(\Pi x : \alpha.(\Pi y : \alpha.t)))$
4. $\approx$ is of category $(\Pi \alpha : \star.(\Pi \beta : \star.t))$
5. $\forall$ and $\exists$ are of category $(\Pi \alpha : \star.(\Pi y : (\Pi x : \alpha.t).t))$
6. $\mathbb{V}$ and $\mathbb{E}$ are of category $(\Pi x : (\Pi \alpha : \star.t).t)$

The Boolean connectives operate similarly in pure-type theory as in simply-typed languages. In each, they take sequences of sentences as their inputs—and have a single sentence as their outputs. However, the identity sign '=' operates somewhat differently. In simply-typed languages, there are infinitely many identity signs; for every type τ there is a constant of type $(\tau \rightarrow (\tau \rightarrow t))$ used to express the claim that terms of type τ are identical. By contrast, the Calculus of Constructions contains a single identity predicate for terms of every type. Moreover, it is a function with a sequence of three inputs, rather than two; the first input is a type, and the subsequent two inputs are terms of that type. The output of this function is a sentence—intuitively, the sentence asserting that the second two inputs are identical. Nevertheless, it would be inaccurate to claim that this language only contains one predicate for identity. While there is but a single identity predicate for terms of arbitrary type, there is a second identity predicate—$\approx$—used to express the identity of the types themselves.[22]

Constructing Purity

In simply-typed languages, we require infinitely many predicates for fundamentality—one for each of the infinitely many types. By contrast, within the Calculus of Constructions we can operate with a single fundamentality predicate: one that applies to terms of every type. Just as identity is a function whose first input is a type and whose subsequent inputs are terms of that type, so too the predicate 'fundamental' is a function whose first input is

[21] For proof, see Barendregt (1992).

[22] Similarly, there is but a single universal and existential quantifier for terms of every type (one which takes a type α as its first input, and a term typed as a function from α to t as its second)—as well as a universal and existential quantifier for the types themselves.

a type and whose subsequent input is a term of that type. More precisely, let us introduce a predicate for fundamentality of the following grammatical category:[23]

$$\mathcal{F} : (\Pi\alpha : \star.(\Pi x : \alpha.t))$$

With the use of this predicate, we restate Strong Purity in a manner that makes quantification over types explicit:[24]

Strong Purity$_3$:
$\forall p^t \mathbb{V}\alpha \forall X^{\Pi w : \alpha.t} \forall x^\alpha.((\mathcal{F}(p) \wedge p = Xx) \rightarrow (\mathcal{F}(X) \wedge \mathcal{F}(x)))$

Weak Purity can then be formalized in the obvious way:

Weak Purity$_3$:
$\forall p^t(\mathcal{F}(p) \rightarrow \mathbb{E}\alpha \exists X^{\Pi w : \alpha.t} \exists x^\alpha.(\mathcal{F}(X) \wedge \mathcal{F}(x) \wedge (p = Xx)))$

Metaphysicians who appeal to either principle ought to be concerned with their consistency. As languages expand their expressive power, the risk of contradiction grows. In order to prove that Strong and Weak Purity$_3$ are consistent, we would need to introduce a model theory for the Calculus of Constructions—one that is notoriously arduous—and establish the existence of a model that validates each principle. I will not discuss this model theory here and so am unable to prove that these principles are consistent.

Nevertheless, I think that we ought to be extremely confident that both Strong and Weak Purity$_3$ are consistent—perhaps as confident as we could possibly be in the absence of proof. Deviant interpretations of our predicate $\mathcal{F}$ are sure to validate both principles. While my intended use of '$\mathcal{F}$' is to denote fundamentality, from a mathematical perspective it could denote any property whatsoever. Suppose we interpret it to mean self-distinctness—so that $\mathcal{F}(x^t) := \lambda x^t.x \neq x$. On this interpretation, both Strong and Weak Purity$_3$ are vacuously true; each is a conditional with a false antecedent, since every proposition is self-identical.[25] If the claim that no proposition is distinct from itself is

[23]Note that this predicate cannot assert that a type itself is fundamental—though we could introduce an alternate predicate $\mathcal{F}_\star$ of category $\Pi\alpha : \star.t$ for this purpose. There are potential theoretical uses for this second fundamentality predicate. We might think of the predicate $\mathcal{F}$ as expressing fundamentality *relative* to a certain type—namely, the type that serves as its first input. We could define an absolute predicate which requires that a term ϕ be fundamental relative to its type α, and also that α is fundamental—that is, iff $\mathcal{F}(\alpha, \phi) \wedge \mathcal{F}_\star(\alpha)$. This alternative introduces another potential disambiguation of Purity (though one I will not explore here); Purity might be interpreted either in terms of relativized or absolute fundamentality—on either the Strong or Weak version.

[24]In what follows, I omit the first input of $\mathcal{F}$, as it is contextually evident.

[25]There are also almost surely non-vacuous models for these principles. We could, alternatively, interpret $\mathcal{F}$ to mean 'is self identical,' so that $\mathcal{F}(x) := \lambda x^\tau.x = x$. This interpretation nearly validates Strong and Weak Purity$_3$ itself; the only further requirement is that every proposition is identical to some-instance-of-predication-or-other. If the Calculus of Constructions permits β-identification—so that we can consistently state that $\lambda x.Fx(a) = Fa$—this will suffice. Note, however, that for the vacuous interpretation we only needed the assumption that all propositions are not distinct from themselves; for the nonvacuous interpretation we need the assumption that terms of arbitrary type are self-identical.

consistent, then Strong and Weak Purity$_3$ are also consistent.

The Return of Simply-Typed Purity

Weak Purity$_3$ strikes me as the most faithful interpretation yet. But some might maintain that even this formulation is too strong. It requires that, for a fundamental fact $[p]$, there exist *immediate* fundamental constituents that generate $[p]$; there must exist two fundamental terms (of some type or other) such that predicating the first of the second is identical to $[p]$. But perhaps $[p]$ is constructed from fundamentalia 'down the line.' That is, it could be that the immediate terms that generate $[p]$ are derivative, but these derivative terms are themselves constructible from purely fundamental terms.[26]

An example might help clarify this thought. Of course, any particular instance of fundamentality is bound to be controversial—there is currently no consensus over what the fundamental facts are—but one can serve to illustrate the structure that I have in mind. Suppose that both electron *e* and the relation of *having the same charge as* are fundamental—and let us denote these with 'e' and '$\lambda x, y.SCxy$' respectively. It seems plausible that the property '$\lambda y.SCey$'—the property of *having the same charge as e*—is derivative. Many objects bear this property due to facts about their microphysical structures; the reason an object has the same charge as an electron is due to the fact that it contains one more electron than proton. Because the bearing of this property is typically explained in virtue of the distribution of various particles, the property is not itself fundamental. Nevertheless, we might maintain that $[SCee]$ is a fundamental fact; nothing explains why e has the same charge as itself. If this is so, then $[SCee]$ is a fundamental fact whose immediate constituents are derivative—that can nevertheless be constructed from fundamentalia.[27]

Weak Purity$_3$ can be modified to allow for mediate—rather than merely immediate—fundamental construction. Interestingly, this modification can be stated in simply-typed languages. The schematic approach used to express Strong Purity$_2$ applies to this principle as well. In the present context, the obvious way to formalize mediate fundamental construction is via recursion.

[26]Under certain additional assumptions, such a case could not arise. If the result of predicating one fundamental term of another always results in something fundamental (e.g., if $(\mathcal{F}(F) \wedge \mathcal{F}(a)) \to \mathcal{F}(Fa)$), fundamentality would never 'leapfrog' over terms in this way. However, the claim that combining fundamental terms always results in something fundamental is not particularly plausible. Even if we grant that [Electron *e* is negatively charged], [Proton *p* is positively charged] and disjunction are all fundamental, we might deny that [Either electron *e* is negatively charged or proton *p* is positively charged] is fundamental. This sort of case is taken from Raven (2016). Quite generally, it seems that we will be able to generate gruesome combinations of fundamental facts—combinations that are not plausibly fundamental themselves.

[27]This example could be resisted in various ways. Aside from denying that either e or $\lambda x.y.SCxy$ are fundamental, a philosopher might claim that $[SCee]$ is itself derivative; perhaps *charge* is a gradable property—and $[SCee]$ is grounded in the fact that every object bears the same gradable property as itself. Alternatively, it could be that there is some other way of constructing this fact from other fundamental properties; perhaps $\lambda x.SCxx$ is fundamental (my thanks to Isaac Wilhelm for this suggestion).

For every type τ, let us introduce a new predicate FC of type $(\tau \to t)$ with the intended interpretation that '$FC(\phi)$' asserts that ϕ is fundamentally constructible. For any terms ϕ of type $(\tau_1 \to \tau_2)$ and ψ of type τ_1, fundamental constructibility is defined as follows:[28]

$$
\begin{array}{ll}
\textit{i.} & (Fundamental(\phi) \wedge Fundamental(\psi)) \to FC(\phi(\psi)) \\
\textit{ii.} & (Fundamental(\phi) \wedge FC(\psi)) \to FC(\phi(\psi)) \\
\textit{iii.} & (FC(\phi) \wedge Fundamental(\psi)) \to FC(\phi(\psi)) \\
\textit{iv.} & (FC(\phi) \wedge FC(\psi)) \to FC(\phi(\psi))
\end{array}
$$

All and only the fundamentally constructible terms are given by these recursive clauses. In effect, terms that result from predicating any sequence of fundamental terms of one another are said to be fundamentally constructible. Take an example: suppose that p, q and $\lambda x, y.x \wedge y$ are all fundamental. Given line *i*, $\lambda y.p \wedge y$ is fundamentally constructible; given line *iii*, the conjunction $p \wedge q$ is as well.

We can formalize Weak Purity in terms of fundamental construction simply as:

Weak Purity$_4$:
$\forall p^t(Fundamental(p) \to FC(p))$

If $[p]$ is fundamental, then $[p]$ is fundamentally constructible.

Thus far, we have progressively weakened the interpretation of Purity; each formulation has required less of the world than the ones that preceded it. However, Weak Purity$_4$ has arguably overshot the mark. Given plausible assumptions about fundamentality and granularity, this principle is trivially true. Moreover, there are numerous paths toward triviality; several combinations of plausible assumptions entail that Weak Purity$_4$ is true.

Suppose, as seems plausible, that the Boolean connectives are fundamental; $\neg$, $\vee$, $\wedge$ and all the rest are fundamental operators.[29] Many accounts of propositional identity license the principle Involution, according to which $p = \neg\neg p$.[30] If this is so, then Weak

[28] In the obvious way, we could express fundamental construction in the Calculus of Constructions by introducing FC as a relation between types and terms—in much the way that we introduced a single predicate for fundamentality within this language.

[29] There are various reasons why philosophers might deny that these connectives are fundamental. Some, like Lewis (1986), maintain that fundamental reality is non-redundant; there are no fundamental facts $[p_1]$ and $[p_2]$ such that $[p_1]$ entails $[p_2]$. The thought underlying non-redundancy requirements is that, when God was creating the world, she was lazy—and did not want to do more work than was needed. Fundamental logical operators would be redundant, as facts about many combinations of them entail facts about the others. For discussions of this point, see Sider (2011).

[30] Coarse-grained theories (according to which necessarily equivalent propositions are identical) license Involution—see e.g., Lewis (1986). Relatedly, Classicists—who maintain that provably equivalent propositions are identical, also endorse Involution—see e.g., Bacon and Dorr (2024). Many fine-grained theories of propositional identity also license this principle—see, e.g., Fine (2017*a*,*b*). For an explicit argument that propositions are their double negations (along the lines that languages whose syntax ensured Involution would not seem to be missing anything about the world) see Ramsey (1927).

Purity$_4$ is true. Take an arbitrary fundamental p. Line *i* entails that $\neg p$ is fundamentally constructible—and line *ii* then entails that $\neg\neg p$ is fundamentally constructible as well. Given Involution, $\neg\neg p = p$; so, by Leibniz's Law, p is fundamentally constructible. Because the selection of p was arbitrary, all fundamental facts are fundamentally constructible.

We need not appeal to Involution to trivialize Weak Purity$_4$. For similar reasons, philosophers who endorse Idempotence—the claims that $p = p \wedge p$ and $p = p \vee p$—are committed to this principle. The (admittedly more controversial) principles of Absorption ($p = p \vee (p \wedge q)$), and Dissolution ($p = p \vee (q \wedge \neg q)$ and $p = p \wedge (q \vee \neg q)$) likewise trivialize this interpretation. Given either principle, an arbitrary fundamental p could be constructed from itself, the logical operators and an arbitrary fundamental q. Nor is it even necessary to appeal to fundamental Boolean connectives. If the operator $\lambda x^t.x$ is fundamental—and if (as previously suggested) β-equivalent terms co-refer—Weak Purity$_4$ is trivially true.[31]

In each of these cases, a fundamental fact $[p]$ can be constructed from fundamentalia—but *it itself* is one of the fundamental constituents that it is built from. Insofar as construction is a process through which some terms are constructed from others, our formalization ought to rule out cases where a term constructs itself. Moreover, trivial versions of Weak Purity are theoretically impotent. While true, they are useless for metaphysical inquiry; one could not use such a principle to uncover novel truths about the fundamental, as they merely reflect the fact that each fundamental fact can construct itself.

It is straightforward to modify the definition of *FC* so as to preclude self-construction.[32] In effect, we introduce a function that 'rules out' individual propositions from figuring within construction: one that asserts that a term can be constructed from fundamentalia other than p (for a given p). To that end, let FC^{-x} be a schematic function with a sequence of two inputs. The first input is always of type t (the term which cannot figure within construction) and the second is a term of arbitrary type (the term which is constructed). As before, the clauses for this function are given recursively:

i. $(Fundamental(\phi) \wedge Fundamental(\psi) \wedge \psi \neq p) \rightarrow FC^{-p}(\phi(\psi))$
ii. $(Fundamental(\phi) \wedge FC^{-p}(\psi)) \rightarrow FC^{-p}(\phi(\psi))$
iii. $(FC^{-p}(\phi) \wedge Fundamental(\psi) \wedge \psi \neq p) \rightarrow FC^{-p}(\phi(\psi))$
iv. $(FC^{-p}(\phi) \wedge FC^{-p}(\psi)) \rightarrow FC^{-p}(\phi(\psi))$

[31] This last suggestion threatens to trivialize not only Weak Purity$_4$ but Weak Purity$_3$ as well. Those who hold that $\lambda x^t.x$ is fundamental ought to index Weak Purity$_3$ in a similar manner to the following indexing of Weak Purity$_4$.

[32] While I myself rule out trivializing self-construction via FC$^-$, an anonymous reviewer points out that the principle *Only Logical Circles* (Dorr (2016)) could perform similar work. According to this principle, if there is a circular identification—$p = \phi$ where p occurs within ϕ—then all other terms within ϕ are logical. If logical terms are not fundamental, such identifications cannot yield fundamental construction, so the shift from FC to FC$^-$ noted below is unneeded.

(Note that there is no need to specify that p is distinct from ϕ, as ϕ functions predicatively within these clauses, and terms of type t are not predicates).

As before, Weak Purity can then be interpreted as:

Weak Purity$_5$:
$\forall p^t(Fundamental(p) \rightarrow FC^{-p}(p))$

Every fundamental fact can be constructed from fundamental terms other than itself.[33]

Perhaps some suspect that trivialization has not yet been avoided.[34] Another route depends upon β-conversion. Thus far, all instances of β-conversion have been nonvacuous: cases where $\phi^{[x/\psi]} = \lambda x.(\phi)(\psi)$, in which x occurs free within ϕ. A more controversial version of this principle concerns *vacuous* β-conversion: cases where x does *not* occur free within ϕ.[35] If terms are identical to their vacuous β-conversions, then every term can be constructed out of any other; for arbitrary terms ϕ and ψ without free variables, $\phi = \lambda x.(\phi)(\psi)$. For this reason, we might suspect that each fundamental fact can be constructed from fundamentalia; given an arbitrary fundamental fact p and fundamental object e, we have that $p = \lambda x.(p)(a)$. While this appears to be yet another instance of self-construction, the terms that generate p are strictly $\lambda x.(p)$ and a, rather than p itself.

This trivialization can be resisted in at least two ways. A metaphysician might maintain that while identity is preserved through nonvacuous β-conversion, it is not preserved vacuously. If this is so, then the identity $p = \lambda x.(p)(a)$ may be false—so cannot witness the claim that there is a way of constructing p from fundamentalia. Alternatively, she could deny the inference from $Fundamental(p)$ to $Fundamental(\lambda x.(p))$; while $[p]$ may be a fundamental fact, the property of *being such that p* may be derivative. If this is so, then the identification $p = \lambda x.(p)(a)$ need not involve constructing p from purely fundamental terms. While the case of vacuous β-conversion threatens triviality, there are ways to avoid it.

I think that Weak Purity$_5$ is the most faithful interpretation yet. It reflects the thought that every fundamental fact can be fundamentally constructed—while precluding trivial self-construction. Strong Purity$_2$ and Strong Purity$_3$ strike me as the most faithful versions of the universal interpretation; they reflect the thought that every method of constructing fundamental facts relies upon fundamental constituents. (The choice between them only turns on the desirability of expressing type-quantification in the object language). Ultimately, this discussion reveals that Purity might mean a number of different things; the

[33]While I myself believe that Weak Purity$_5$ is adequate, there is room for further refinement. My own appeal to FC^- is to rule out trivializing self-construction. However, I note that this function could be used to exclude other facts from serving in constructions as well. For example, some philosophers might want to rule out the conjunctive parts of a fact from serving as its fundamental constituents (perhaps adopting the truth-maker approach to conjunctive parthood—see Fine (2017*a*,*b*)). Once conditions a fact must meet to be excluded are formalized, it is straightforward to use FC^- to preclude constructions involving those facts in the interpretation of Weak Purity.

[34]My thanks to Peter Fritz for suggesting this to me.

[35]For a discussion of vacuous β-identification, see Goodman (2024).

principle that fundamental facts have fundamental constituents allows for stronger and weaker interpretations. Given this ambiguity, it is incumbent to determine how Purity figures within philosophical argumentation.

Appeals to Purity

Purity is intended to have broad scope. It applies to all facts whatsoever—regardless of their content. While it could be (and has been) cited in any number of contexts, arguments that employ it often have a similar structure. Consider two examples: theories of iterated ground and the grounds of identification.

Iterated Ground

Suppose that Chris is experiencing phenomenal pain—and the fact that Chris is in pain is grounded in the fact that his C-fibers are firing.[36] Why is Chris's pain grounded in this way? Why does phenomenal pain correspond to firing C-fibers, rather than some other neurological configuration? More generally, what explains why the grounding relation obtains in the cases that it does? What grounds facts about grounding?

These questions impact traditional philosophical debates. Philosophers armed with a theory of iterated ground have the resources to explain, in a deep sense, what makes their theories true. For example, a normative naturalist who understands the grounds of grounding can explain not only the natural foundations of normative facts—but also why normative facts are grounded in the way that they are.

Some might maintain that the grounding facts are fundamental. These facts seem to be natural stopping points for metaphysical explanation; if the grounding relation itself is primitive, there may be no reason why it obtains in some cases but not others. Nevertheless, many philosophers deny that the grounding facts are fundamental—some of whom cite Purity.[37] Take the following:

> "Here is a truth: there exists a city. Since the notion of a city is not fundamental, purity says that this truth is not fundamental...this truth holds in virtue of some fundamental truth *T*–perhaps some truth of microphysics. So we have:
>
> (1) There is a city in virtue of the fact that *T*

[36]Some in this area do not interpret Purity in terms of grounding. For ease of prose, I describe these cases in terms of ground, but this locution could easily be switched for any other that a metaphysician prefers.

[37]Philosophers who appeal to this argument include Sider (2011, 2020), Bennett (2011), deRosset (2013, 2023), and Dasgupta (2014*b*). This is not the only motivation for theories of iterated ground. Another—which appeals to principles of free modal recombination, rather than Purity, occurs in Schaffer (2010) and Bennett (2011). Yet another, which appeals to a principle of settledness, occurs in Correia (2023).

...But now consider (1) itself. Just like 'There are cities,' (1) is a truth involving the notion of a city. And so, given purity, it cannot be a fundamental truth...Purity...requires facts about the *relationship* between the fundamental and the nonfundamental to be themselves nonfundamental. Thus purity brings a heavy explanatory burden: it requires there to be facts in virtue of which in-virtue-of-facts hold." (Sider, 2011, p. 107)

It is straightforward to reconstruct Sider's argument. Let us denote the grounds of cityhood—whatever those may be—as 'the fact that T'. There is, then, a fact:

[There is a city in virtue of the fact that T]

We can construct this fact from the property of *being a city*; in particular, this fact results from predicating $\lambda X^{e \to t}$.*there is an X in virtue of the fact that T* of *is a city*. According to Purity, fundamental facts only have fundamental constituents. Since we can construct this fact from *being a city*, the property of *being a city* is one of its constituents—and is therefore fundamental if [There is a city in virtue of the fact that T] is fundamental. But the property of *being a city* is *not* fundamental. Therefore, the fact [There is a city in virtue of the fact that T] is not fundamental.

The Grounds of Identification

Consider the fact:

[Hesperus = Phosphorus]

Why is this so? Some philosophers think that the answer is: there is no explanation; identifications are metaphysically fundamental. As (Dorr, 2016, p. 41) says, "Identities are excellent stopping places for explanation; they do not cry out for explanation in their own right. Indeed, there is something odd about questions like 'Why is Hesperus Phosphorus?'. Unless this is understood as a request to be reminded of the reasons for believing that Hesperus is Phosphorus, it is hard to know what would count as a satisfying answer."[38]

Nevertheless, a growing number of philosophers deny that identifications are fundamental, many citing Purity.[39] Take the following:

[38] Others who deny that identity can be metaphysically explained include Lewis (1986), Salmon (1987), and Horsten (2010).

[39] Aside from Shumener (2019), examples of philosophers who appeal to an argument of this sort include Litland (2023), Rubenstein (2023), and Elgin (2024*a*). A philosopher who hints at—but does not explicitly endorse—this argument is Wilhelm (2020). Not all philosophers who maintain that identifications are derivative appeal to this argument—e.g., Fine (2016) argues that identifications are zero-grounded (but may have substantive grounds) without appeal to Purity.

> "The identity problem arises when we look to metaphysically explain the identity facts involving concrete objects, facts like The Rock of Gibraltar = The Rock of Gibraltar, and The Original McDonalds Big Mac = The Original McDonalds Big Mac...many such identity facts strike us as non-fundamental: they often involve non-fundamental objects like Big Macs and giant rocks, and it is doubtful that fundamental facts should involve non-fundamental objects." (Shumener, 2019, p. 2074)

Shumener does not mention Purity by name, but it underlines her argument nonetheless. Take an identification—for example, [The Rock of Gibraltar = The Rock of Gibraltar]. We can construct this fact from *The Rock of Gibraltar*; in particular, it results from predicating self-identity ($\lambda x^e.x = x$) of *The Rock of Gibraltar.* According to Purity, fundamental facts only have fundamental constituents. Because we can construct this fact from *The Rock of Gibraltar*, the rock is one of its constituents—and is therefore fundamental if [The Rock of Gibraltar = the Rock of Gibraltar] is fundamental. But *The Rock of Gibraltar* is *not* fundamental. Therefore, the fact [The Rock of Gibraltar = The Rock of Gibraltar] is not fundamental.

Generalized Appeals to Purity

Purity has been cited in numerous other contexts. It arises in discussions of the grounds of certain modal facts (e.g., [Necessarily, all cities are cities]), negative facts (e.g., [It is not the case that there is a direct flight between Australia and Spain]), and nongrounding facts (e.g., [The fact that Socrates is wise is not grounded in the fact that {Socrates} contains someone wise]).[40]

In each of these cases, a philosopher argues as follows (for a given fact $[p]$ and entity e in some domain of interest):

i. e is a constituent of $[p]$.
ii. e is not fundamental.
iii. Purity: fundamental facts only have fundamental constituents.
iv. Therefore, $[p]$ is not fundamental.

Arguments of this structure are only valid on Strong interpretations of Purity. If every method of constructing a fundamental fact $[p]$ involves fundamental constituents—and if one method of construction involves entity e, then e must be fundamental; if we deny that e is fundamental, we ought to deny that $[p]$ is fundamental as well. By contrast, Weak Purity—in its various indices—does not justify this inference. If there is some-method-or-other of constructing $[p]$ from fundamental constituents, and if one method of construction

[40]For discussions of modal and negative facts, see Sider (2011)—for a discussion of nongrounding facts see Elgin (Forthcoming) and Rumm (Forthcoming).

involves entity e, this does not entail that e is fundamental. This style of argument depends upon Strong Purity and not merely Weak Purity.[41]

Purity in Argumentation

I think that we ought to be skeptical of Strong Purity. There are two reasons for my skepticism. Plausible metaphysical positions are compatible with Weak but not Strong Purity. Moreover, the positive arguments given for Purity (limited though they are) only justify the Weak interpretation.

Conflict with Plausible Views

A dominant theory of identification is *Classicism*, according to which provably equivalent terms are identical: if $\vdash p \leftrightarrow q$, then $p = q$.[42] Classicism is a simple and elegant theory of identification. Classicists who endorse Strong Purity must maintain that everything is fundamental.[43] Take a fundamental fact $[p]$ and an arbitrary object a. Provably, $[p] \leftrightarrow [\lambda x.(p \vee (Fx \wedge \neg Fx)(a)]$. $[p]$ is equivalent to the fact that $[a]$ bears *being such that either p is true, or a is both F and not F*. Given Classicism, it follows that $[p] = [\lambda x.(p \vee (Fx \wedge \neg Fx)(a)]$. Therefore, fundamental $[p]$ can be constructed from a. If Strong Purity is true, it follows that a is fundamental—and, since the selection of a was arbitrary, that absolutely everything is fundamental.

This does not hold for Weak Purity. While a fundamental $[p]$ can be constructed from an arbitrary a, this does not entail that a is fundamental, so long as there is some way to construct $[p]$ from fundamentalia other than a. Classicists who would avoid universal fundamentality ought to reject Strong Purity, but need not reject Weak Purity.

We need not adopt Classicism to face this sort of problem. Those who endorse vacuous β-identification—the claim that $\phi = \lambda x.(\phi)(\psi)$ where x is not free within ϕ—must also maintain that everything is fundamental.[44] Vacuous β-identification entails that every term can be constructed from any other. So, given a fundamental fact $[p]$ and an arbitrary term ϕ, $[p]$ can be vacuously constructed from ϕ. On Strong interpretations of Purity, this entails that ϕ is fundamental. Because the selection of ϕ was arbitrary, absolutely everything is fundamental.

Another view that rules out Strong Purity concerns the kinds of things that are fundamental. Some maintain that fundamental reality consists of objects and properties that

[41]This is not to say that these arguments universally succeed if Strong Purity is true—there are any number of other places that they might fail. But for us to be justified in appealing to them, we need a reason to think that Strong Purity holds. Weak Purity is not enough.

[42]See Bacon and Dorr (2024).

[43]More precisely, they must maintain that either everything is fundamental or nothing is—but I set aside the possibility that nothing is fundamental for the purposes of this paper.

[44]My thanks to Peter Fritz for pointing this out to me.

those objects bear; all fundamental facts concern what objects there are and how those objects are. On this view—sometimes called 'Truth Supervenes on Being'—fundamental facts involve fundamental objects and properties. There are no fundamental terms of other types. This sort of position has been used to argue against the fundamentality of counterfactuals, totality facts and negative facts.[45]

Arguments for this view would take us far afield. For my purposes, what matters is that *if this is true*, then nearly all theories of propositional identity are incompatible with Strong Purity. As mentioned previously, many license Involution: $[p] = [\neg\neg p]$. For this reason, a fundamental fact $[p]$ can be constructed from negation—$[p] = [\lambda x^t.(\neg x)(\neg p)]$. If Strong Purity is true, it follows that negation is fundamental. But, according to Truth Supervenes on Being, negation is *not* fundamental. There are no fundamental sentential operators on this view, and negation is a sentential operator. Therefore, this view rules out Strong Purity (on the assumption that Involution is true).[46] Notably, it does not rule out Weak Purity; so long as $[p]$ is constructible from other fundamental terms, negation need not be fundamental.

To be clear, I do not endorse Classicism, vacuous β-identification, or the claim that Truth Supervenes on Being here. They strike me as plausible—yet controversial—metaphysical commitments. But it is telling that eminently defensible views rule out Strong Purity, without ruling out Weak Purity.

Arguments for Purity

There are remarkably few arguments for Purity given its methodological role. Philosophers often treat it as a starting point for inquiry. However, there are two sorts of motivations typically given: functional and definitional.

The most common argument depends upon the theoretical role that fundamentality plays. While fundamental facts may stand in grounding relations, fundamental things serve to construct fundamental facts. As such, it ought to be possible to describe fundamental reality without reference to derivative things. While this characterization (in terms of reference) is given semantically, the underlying point is worldly. As Sider (2011) says, "When God was creating the world, she was not required to think in terms of nonfundamental notions like city, smile, or candy." (pg. 106).

This argument only supports Weak Purity. This becomes apparent when we consider an expressively impoverished language: one that lacks any constants for terms of type t. Within this language, the only way to express a fact is to express elements that compose it (for example, this language may have an expression of the form 'Fa' but not 'p'). Moreover, the only constants of other types denote fundamental terms (so there is a predicate 'F'

[45] For a discussion of this kind of view, see Sider (2011).

[46] In an analogous way, this view is incompatible with Idempotence—$p = p \wedge p$ and $p = p \vee p$. This result reflects a more general feature of Strong Purity. If it holds, then on nearly every account of propositional identity $\neg$, $\wedge$ and $\vee$ are fundamental.

within this language just in case the semantic value of '*F*' is a fundamental property). Would it be possible to express every fundamental fact within this language? If Weak Purity is true, arguably, yes. There is some way or other of constructing each fundamental fact from fundamental things—and we can use this method of construction to express all of fundamentality. And, metaphorically, God could have constructed the fundamental facts purely from fundamental constituents by not appealing to the methods of construction involving the derivative. More generally, the fundamental can serve its intended role as the building blocks of fundamental facts even if Strong Purity is false. Weak Purity ensures that every fundamental fact can be constructed from purely fundamental constituents—which is what Purity was intended to ensure.

The other sort of argument is definitional; metaphysicians define the phrase 'fundamental fact' so as to refer to a fact with purely fundamental constituents.[47] While metaphysicians do not typically specify whether this definition involves Strong or Weak Purity, there seems to be nothing to prevent them from specifying that their use of 'fundamental fact' in Strong terms. On their use of 'fundamental,' the inference from '*e* is a constituent of a fundamental fact' to '*e* is fundamental' is valid—so it might seem that these philosophers avoid the problem that I raise.

I do not object to philosophers defining terms as they wish; 'fundamental' is a quasi-technical term, and there is nothing wrong with a philosopher specifying what they mean by it. However, there are a few reasons to dispute this as a defense of Purity.

First, philosophers can always define terms in a way that trivializes substantive questions. A metaphysician can assert 'by 'fundamental' I mean 'a property borne by electrons''—after which there is no longer a substantive debate over whether electrons are fundamental (at least on their use of the term). While we could define 'fundamental' in a manner that verifies Strong Purity, this has nothing to do with Purity itself; any controversial debate could be settled by stipulative definition in this way.

Such philosophers may also be saddled with uncomfortable metaphysical commitments. Even if they define 'fundamental' as they would like, they do not avoid the conflict with Classicism, vacuous β-identification, and the view that Truth Supervenes on Being noted above.

Moreover, it is not entirely clear that fundamentality can satisfy other theoretical demands if it validates Strong Purity by definition. For example, some philosophers in this area also subscribe to the principle of Completeness, according to which every derivative truth can be constructed from fundamental truths.[48] Suppose there were a fact $[p]$ which can be constructed from fundamental truths, but not every method of construction relies upon fundamentalia (the sort of fact that was thought to undermine Strong Purity). On the Strong definition of 'fundamental,' this fact is not fundamental itself. So, $[p]$ cannot

[47] Take, e.g., "Say that a fact is fundamental (or brute) if it does not obtain in virtue of other facts, and that a thing is fundamental if it is a constituent of a fundamental fact" (Rosen, 2010, p. 112). Another argument by definition occurs within deRosset (2023).

[48] See Sider (2011), Eddon (2013), Tahko (2021), and Wilhelm (Forthcoming*a*).

partially validate Completeness; if a fact $[q]$ can be constructed from $[p]$, this does nothing to support the claim that all truths can be constructed from fundamental truths, as $[p]$ is not fundamental on this definition.[49] The worry is that the Strong definition of Purity is so demanding that there will not be sufficiently many fundamental facts to satisfy other principles that the fundamental is meant to satisfy.

Buttressing Purity

Perhaps some suspect that Purity's ambiguity is not as philosophically significant as I make out.[50] I argue that arguments that appeal to Purity are invalid on the Weak interpretation—but any invalid argument can be made valid by including an additional premise. Perhaps adherents of Purity could appeal to arguments of the following form:

i. e is a constituent of $[p]$.
ii. e is not fundamental.
iii. Weak Purity: if a fact $[p]$ is fundamental, then some way of constructing $[p]$ has only fundamental constituents.
iv. Fundamental Uniformity: if there is some way of constructing $[p]$ from only fundamental constituents, then every way of constructing $[p]$ has only fundamental constituents.
v. Therefore, $[p]$ is not fundamental.

This modified argument is valid. If Weak Purity holds, it follows that $[p]$ is not fundamental if Fundamental Uniformity also holds. After all, if there is no way to construct $[p]$ from only fundamental constituents, then even Weak Purity entails that $[p]$ is not fundamental. So, if we have a compelling reason to endorse Fundamental Uniformity, we need not be concerned about the distinction between Strong and Weak Purity.

The problem is that we lack a compelling reason to endorse Fundamental Uniformity. This principle cannot be defended by assuming that identical facts have identical constituents; we cannot suppose that if there is one way to construct $[p]$ from only fundamental constituents, then every way of constructing $[p]$ involves those same constituents. As has already been belabored, the claim that identical facts have identical constituents is inconsistent in any logic with either β-identification or the ability to engage in Russell-Myhill reasoning.

Moreover, there are independent reasons to think that Fundamental Uniformity is false. For the sake of concreteness, let us return to the case of identification: [The Rock

[49] Note that we cannot assume that it will be possible to construct these facts from whatever fundamentalia construct $[p]$ in a manner that validates Completeness. Completeness is a principle concerning building *facts*—terms of type t—from one another, and there is no guarantee that the fundamental terms from which $[p]$ are constructed are of type t.

[50] My thanks to Trevor Teitel and an anonymous reviewer for raising this objection.

of Gibraltar = The Rock of Gibraltar]. Clearly, one way to construct this fact involves a derivative entity; this is the fact that results from predicating self-identity—$\lambda x.(x = x)$—of the Rock of Gibraltar, and the Rock of Gibraltar is not plausibly fundamental. But there may be ways of constructing this fact that do not appeal to the derivative.

In fact, β-identification itself entails that this fact can be constructed without appeal to the Rock of Gibraltar. This fact also can be constructed by predicating $\lambda X.(X(RoG))$ of $\lambda x.(x = x)$—i.e., by predicating the second-order property *is a property of the Rock of Gibraltar* of the first-order property *is self-identical*. Neither one of these terms is identical to the Rock of Gibraltar (after all, the Rock of Gibraltar is an object, which neither of these is) yet predicating the first of the second results in [The Rock of Gibraltar = The Rock of Gibraltar]. Because this fact is constructible from terms other than the Rock, even if we grant that the Rock is derivative, this does not guarantee that terms other than the Rock that construct this fact are derivative.

Some might suspect that, while this alternate method of construction involves terms other than the Rock, these terms are derivative themselves. After all, if the Rock is not fundamental, then *being a property of the Rock* may not be fundamental either. But there may be ways to construct this fact from yet other terms: ones that are plausibly fundamental. This is certainly so on coarse-grained conceptions of facts. If all necessarily equivalent facts are identical, then, given the necessity of identity, [The Rock of Gibraltar = The Rock of Gibraltar] is identical to every other necessary truth. In particular, it is identical to $[e = e]$ for an arbitrary fundamental entity e. So, it is possible to construct this fact both from a derivative entity (the Rock of Gibraltar) or a fundamental entity (e) and Fundamental Uniformity is false.

We need not appeal to coarse-grained conceptions of facts. Let us suppose that the Rock is identical to the mereological fusion of simples $s_1, s_2, ..., s_n$—and let us represent this fusion with $f(s_1, s_2, ..., s_n)$. Given this identification, Leibniz's Law entails that [The Rock of Gibraltar = The Rock of Gibraltar] = $[f(s_1, s_2, ..., s_n) = f(s_1, s_2, ..., s_n)]$; the fact that the Rock is self-identical is the same as the fact that this fusion of simples is self-identical. Quite plausibly, both the relation of mereological fusion and the mereological simples are fundamental. If this is so, then there is one way of constructing this fact from a derivative term (the Rock) and a way of constructing this fact from fundamental terms (fusion and simples)—and Fundamental Uniformity is therefore false.

This particular case suggests a general method for generating fundamental construction. For a given fact $[p]$ that can be constructed from a derivative term, this term can be replaced with the fundamental terms that compose it. In the case of concrete objects, a singular term can be replaced by its mereologically simple parts. Once this is done, the resulting fact is constructed from only fundamental constituents—satisfying the demands of Weak Purity (and falsifying Fundamental Uniformity).

The upshot is this. Appeals to Purity could be salvaged by appealing either to Strong Purity, or both Weak Purity and Fundamental Uniformity. However, both Strong Purity and Fundamental Uniformity are—at present—not justified, and there are substantive reasons to think that each is false. As a result, appeals to Purity in argumentation are unjustified.

Conclusion

The principle of Purity—according to which fundamental facts only have fundamental constituents—contains a subtle ambiguity. It could either be interpreted as the claim that every method of constructing a fundamental fact relies upon fundamental constituents or, alternatively, that there exists at least one way of constructing each fundamental fact from fundamental constituents. There are various ways of formalizing these principles, some of which can be expressed in simply-typed languages—and others of which require more expressive power. The arguments that metaphysicians typically use involving Purity are only valid if Strong Purity is true; Weak Purity is not sufficient for their purposes. Nevertheless, Strong Purity is open to doubt. Various defensible philosophical commitments are compatible with Weak—but not Strong—Purity, and the functional argument for Purity can be correct even if only Weak Purity is true. As such, metaphysicians who would appeal to Strong Purity owe an argument: a reason to think that the stronger version holds as well.

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